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Nonlinear Methods for Analyzing the Statistical Dynamics of Tracking Systems in Radio Receivers.

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ABSTRACT

The paper discusses an assessment of the critical ratios (required front-to-back ratios) of signal to noise, which enable the functioning of radio receiving devices (receivers) in environments where there is a deficit of interference immunity with a probability of at least 0.9. The assessment has been performed using Markov's chain (processes) on the basis nonlinear methods for analyzing the statistical dynamics of tracking systems and the theory of optimal filtering of data processes. Most areas of modern science, including biological and medical research, have embraced the usage of remote sensing systems which are based on data gathered by numerous specialized satellites. It is thus necessary not only to understand how these systems work, but suggest methods and techniques for their perfection. This paper is dedicated to advances in this multidisciplinary field, which can later be used to develop remote sensing systems for the purpose of Earth and other sciences.
Keywords: adaptive radio receivers, interference immunity, tracking, optimal algorithm, synthesis.

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INTRODUCTION

Many areas of modern science have found the widest application for data collected by navigation satellite systems, otherwise known as satellite remote sensing. This includes various fields in Earth sciences, biology, and even medical sciences. However many of those researchers specializing in the aforementioned areas have little understanding of how these systems operate and, least of all offer methods for their perfection. In this paper we shall discuss the problems of analyzing statistical data from tracking systems and suggest models for their improvement. Further understanding of the topic may result in new ideas arising from specialist not related to electronics, however having new visions for the application of remote sensing in their particular field of studies.

Modern ground-based command and measuring systems for managing the flight of spacecraft, besides their standard task of receiving command and program data and telemetry, are often required to measure navigational parameters (pseudodistance and pseudospeed) via a received useful signal for the purpose of ephemeris provision [1]. In order to solve this task it is necessary to construct a device capable of tracking the parameters of the received signal with a high degree of accuracy. Measurable parameters for navigational support include time delay and phase. Both parameters are tracked using delay tracking systems (DTS) and phased automatic frequency adaptors (PLL).

The measurement accuracy of these parameters and the possibility of tracking them strongly depend on the interference immunity of the radio receiver.

PROBLEM STATEMENT AND METHODS

Let us evaluate the immunity ratio of signal to interference, which enable radio receivers to operate in conditions of a deficit of interference susceptibility with a probability of less than 0.9.

The time and probability characteristics of the failure of the signal tracking process for a radio receiver it determined using nonlinear methods for analyzing the statistical dynamics of the tracking systems; these methods are based on the Markov chain [2, 3, 4]. The application of these methods allows us to determine the algorithm for calculating the average time and time dispersion to the tracking failure simultaneously for both PLL and DTS considering coupling. Simultaneous signal tracking failure in DTS and PLL is understood as the first exit of the trajectory of the phase φ and delay τ_p beyond the limits of the discrimination characteristics.

FINDINGS

According to [4, 5, 6] a joint statistical dynamics for DTS and PLL can be viewed in the coordinates $\varphi = \hat{\varphi} - \varphi$; $\tau_p = \hat{\tau} - \tau$,

where φ is the phase difference; τ_p is the difference of the time position of the base and input signals; $\hat{\varphi}, \hat{\tau}$ are their values.

The characteristics of tracking failure are determined using differential equations:

$$\begin{aligned}
\frac{d\varphi}{dt} &= -\alpha\varphi - \Delta\varphi\left[\sin 2\varphi + \frac{4\cos\varphi}{AT_0\left(1-\frac{|\tau_p|}{\tau_э}\right)}n_1(t) + \frac{4\sin\varphi}{AT_0\left(1-\frac{|\tau_p|}{\tau_э}\right)}n_2(t) + \frac{8}{A^2T_0^2\left(1-\frac{|\tau_p|}{\tau_э}\right)^2}n_1(t)n_2(t)\right] - n_3(t); \\
\frac{d\tau_p}{dt} &= -\beta\tau - \Delta\tau\left[F(\tau_p) + \frac{2F(\tau_p)}{AT_0\left(1-\frac{|\tau_p|}{\tau_э}\right)\cos\varphi}n_2(t) + \frac{2\tau_э}{AT_0\cos\varphi}n_3(t) + \frac{4\tau_э}{A^2T_0^2\left(1-\frac{|\tau_p|}{\tau_э}\right)\cos^2\varphi}n_2(t)n_3(t)\right] - n\tau(t)
\end{aligned} \qquad (1).$$

where N_0 is the spectral density of the interference level; A is signal amplitude; $\Delta\varphi$ is the synchronization (locking) band of the PLL system; $\Delta\tau$ is the locking band of the DTS; $\tau_э$, T_0 is the duration of the impulse of the pseudorandomness and, accordingly, the duration of the data symbol.

$$\Delta\varphi = \frac{A^4 T_0 \left(1-\frac{|\tau_p|}{\tau_э}\right)^2}{2N_0^2}\bar{k}_{11};\ \Delta\tau = \frac{A^4 T_0 \cos^2\varphi \left(1-\frac{|\tau_p|}{\tau_э}\right)}{2N_0^2 \tau_э}\bar{k}_{22};$$

$$n_1(t) = \int_{(k-1)\cdot T_0}^{k\cdot T_0} n_p(t) g(t-\hat{\tau}) \sin\hat{\Phi} dt;\ n_2(t) = \int_{(k-1)\cdot T_0}^{k\cdot T_0} n_p(t) g(t-\hat{\tau}) \cos\hat{\Phi} dt;$$

$$n_3(t) = \int_{(k-1)\cdot T_0}^{k\cdot T_0} n_p(t) \frac{dg(t-\hat{\tau})}{d\hat{\tau}} \cos\hat{\Phi} dt$$

demonstrate the noises which are normal random processes;

g(t) is the pseudorandom sequence of impulses;

F(τ_p*)* is a function which characterizes the discriminate feature of the DTS; it has the following view

$$F(\tau_p) = \frac{\tau_э}{T_0} \int_{(k-1)\cdot T_0}^{k\cdot T_0} g(t-\tau) \frac{dg(t-\hat{\tau})dt}{d\hat{\tau}} = \frac{\tau_э}{T_0} \int_{(k-1)\cdot T_0}^{k\cdot T_0} g(t-\tau) \frac{g\left(t-\hat{\tau}+\frac{1}{2\tau_э}\right) - g\left(t-\hat{\tau}-\frac{1}{2\tau_э}\right)}{\tau_э} dt.$$

After a piecewise linear approximation, the function *F(*τ_p*)* can be viewed as such

$$F(\tau_p) = \begin{cases} 1-\frac{|\tau_p+2\tau_э|}{\tau_э} & \text{при } -\frac{3}{2}\tau_э < \tau_p < -\frac{1}{2}\tau_э; \\ \frac{\left|\tau_p-\frac{1}{2}\tau_э\right|}{\tau_э} - \frac{\left|\tau_p+\frac{1}{2}\tau_э\right|}{\tau_э} & \text{при } -\frac{1}{2}\tau_э < \tau_p < -\frac{1}{2}\tau_э; \\ \frac{\left|\tau_p-\frac{1}{2}\tau_э\right|}{\tau_э} - 1 & \text{при } -\frac{1}{2}\tau_э < \tau_p < -\frac{3}{2}\tau_э; \\ 0 & \text{при } -\frac{3}{2}\tau_э > \tau_p > \frac{3}{2}\tau_э; \end{cases}$$

For equation (1) we have considered the product of the accepted oscillation *ξ1(t)* and the base signal can be shown as

$$\xi_1(t) g(t-\hat{\tau}) \cos(\omega_0 t + \hat{\psi}) = \frac{1}{2} A \left(1-\frac{|\tau_p|}{\tau_э}\cos\varphi\right) + n_p(t) g(t-\hat{\tau}) \cos(\omega_0 t + \hat{\psi});$$

$$\xi_1(t) g(t-\hat{\tau}) \sin(\omega_0 t + \hat{\psi}) = \frac{1}{2} A \left(1-\frac{|\tau_p|}{\tau_э}\sin\varphi\right) + n_p(t) g(t-\hat{\tau}) \sin(\omega_0 t + \hat{\psi});$$

$$\xi_1(t) \frac{dg(t-\hat{\tau})}{d\hat{\tau}} \cos(\omega_0 t + \hat{\psi}) = \frac{1}{2} A g(t-\tau) \frac{dg(t-\hat{\tau})}{d\hat{\tau}} \cos\varphi + n_p(t) \frac{dg(t-\hat{\tau})}{d\hat{\tau}} \cos(\omega_0 t + \hat{\psi});$$

Besides, during time T_0 the variable φ и τ_p it should be noted that the constant time for PLL and DTS in operating electronics is typically greater than the duration of the information symbol.

Note that *F(*τ_p*)* is wider than the dynamical characteristics of the DTS $F_{дин}$, which is described by the expression

$$F_{дин} = 1 - \frac{|\tau_p|}{\tau_э} F(\tau_p), \tag{2}$$

the limits of which correspond to point $\tau_p = \pm\tau_э$.

The dispersion of random the processes $n_1(t)$, $n_2(t)$ and $n_3(t)$ is determined using the following formulas: $k_{n1n2} = k_{n1n3} = k_{n2n3} = 0$,

where $\sigma_{n_1}^2$, $\sigma_{n_2}^2$, $\sigma_{n_3}^2$ are respective to the dispersion of the random processes $n_1(t)$, $n_2(t)$ and $n_3(t)$; k_{n1n2}, k_{n1n3}, k_{n2n3} are the functions of their mutual correlation.

The differential equations (1) fully describe the statistical dynamics of the tracking systems in a radio receiver for noise-like signals possessing inverse modulation with phase fluctuation and signal delay in an additive noise environment. For time intervals $t_u \geq \Delta\tau$, where $\Delta\tau >> \tau_K$; $\tau_K = T_0$ is the random processes correlation time $n_1(t)$, $n_2(t)$ and $n_3(t)$, equation (1) describes a bidimensional Markov chain (φ, τ_p), the deviation coefficient and diffusion of which are determined by the following formulas

$$a(\lambda_i) = \frac{M\left[\lambda_i(t+\Delta\tau) - \lambda_i(t)\right] | (\lambda_i\lambda_j)}{\Delta\tau};$$
$$b(\lambda_i\lambda_j) = \frac{M\left[\lambda_i(t+\Delta\tau) - \lambda_i(t)\right]\left[\lambda_j(t+\Delta\tau) - \lambda_j(t)\right] | (\lambda_i\lambda_j)}{\Delta\tau}; \tag{4}$$

where $\lambda_1 = \varphi$; $\lambda_2 = \tau_p$; $i = 1,2$; $j = 1,2$.

Using a technique for calculating the deviation coefficient and the diffusion of the Markov chain [7] and considering (3), we will get for equation (1):

$$a(\varphi) = -\alpha\varphi - \Delta\varphi \sin 2\varphi; \qquad a(\tau_p) = -\beta\tau_p + \Delta\tau F(\tau_p);$$
$$b(\tau_p, \tau_p) = \frac{2\Delta^2\tau\delta}{\cos^2\varphi\left(1 - \frac{\tau_p}{\tau_э}\right)^2}\left[F^2(\tau_p) + 2\left(1 - \frac{\tau_p}{\tau_э}\right)^2 + \frac{2\delta}{T_0\cos^2\varphi}\right] + \frac{1}{2}N\tau;$$
$$b(\varphi, \varphi) = \frac{8\Delta^2\varphi\delta}{\left(1 - \frac{\tau_p}{\tau_э}\right)^2}\left[1 + \frac{\delta}{T_0\left(1 - \frac{\tau_p}{\tau_э}\right)^2}\right] + \frac{1}{2}N_\varphi; \tag{5}$$
$$b(\varphi, \tau_p) = \frac{4\sin\varphi\Delta\varphi\Delta\tau F(\tau_p)\delta}{\left(1 - \frac{\tau_p}{\tau_э}\right)^2\cos\varphi},$$

where $\delta = \frac{N_0}{A^2}$ is the ratio of the spectral noise density to the square of the signal amplitude.

The coefficients in equation (5) enable us to determine the probability and time characteristics for signal tracking failure in both DTS and PLL simultaneously; this is performed on the basis of the numerical solution for Pontryagin's second equation [3, 4, 5], which relative to k-th moment of t_k time of the first advance of the phase trajectory and delay of the work zone limits signal

$|\varphi| = \frac{\pi}{2}$, $|\tau_p| = \tau_э$ from the starting point $(\varphi_0, \tau_{p0}) \in \{|\varphi| \leq \pi/2, |\tau_p| \leq \tau_э\}$ will become

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$$\left\{\frac{4\Delta^2\varphi\delta}{\left(1-\frac{|\tau_{p_0}|}{\tau_э}\right)^2}\left[1+\frac{\delta}{T_0\left(1-\frac{|\tau_{p_0}|}{\tau_э}\right)^2}\right]+\frac{1}{4}N_3\right\}\frac{d^2t_k}{d\varphi_0^2}+$$

$$+\left\{\frac{\Delta^2\tau\delta}{\cos^2\varphi_0\left(1-\frac{|\tau_{p_0}|}{\tau_э}\right)^2}\left[F^2\left(\tau_{p_0}\right)+2\left(1-\frac{|\tau_{p_0}|}{\tau_э}\right)^2+\frac{2\delta}{T_0\cos^2\varphi_0}\right]+\frac{1}{4}N\tau\right\}\times$$

$$\times\frac{d^2t_k}{d\tau_{p_0}^2}+\frac{4\sin\varphi_0\Delta\varphi\Delta\tau F(\tau_{p0})\delta}{\left(1-\frac{|\tau_{p_0}|}{\tau_э}\right)^2\cos\varphi_0}\cdot\frac{d^2t_k}{d\varphi_0 d\tau_{p_0}}-\left(\alpha\varphi_0+\sin 2\varphi_0\right)\frac{dt_k}{d\varphi_0}-\left[\beta\tau_{p_0}-\Delta\tau F\left(\tau_{p_0}\right)\right]\frac{dt}{d\tau_{p_0}}=-kt_{k-1},$$

$$(t_0 = 1,\ k = 1,2,...,t_k = 0 \text{ if } |\varphi_0| = \pi/2,\ |\tau_{p0}| = \tau_э) \qquad (6).$$

According to (6), the calculation of the probability and time characteristics of the signal tracking failure incorporates the following expressions that characterize the equivalent PLL and DTS bands

$$f_ш\varphi=\Delta\varphi+\alpha/2; \qquad f_ш\tau=\frac{\Delta\tau}{2\tau_э}+\frac{\beta}{2}, \qquad (7).$$

where $f_ш\varphi$ and $f_ш\tau$ are the equivalent PLL and DTS bands.

If we substitute in (7) the values $\Delta\varphi, \Delta\tau, \overline{k}_{11}, \overline{k}_{22}$ and considering the fact these expressions have been determined for the lesser values φ и τ_p, we will get

$$f_ш\tau=\frac{1}{2}\beta\sqrt{1-\frac{\overline{\nu}_{222}N\tau}{2\beta^2}}; \qquad f_ш\varphi=\frac{1}{2}\alpha\sqrt{1-\frac{\overline{\nu}_{211}N\psi}{2\alpha^2}};$$

$$\overline{\nu}_{211}=-\frac{A^4T_0}{N_0^2}; \qquad \overline{\nu}_{222}=-\frac{A^4T_0}{N_0^2\tau_э^2} \qquad (8).$$

In order to bring the discussed optimal PLL and DTS models in line with existing tracking systems, it is desirable during calculating the probability and time characteristics of tracking failures to fix their equivalent bands at a level, which provides maximum interference immunity of the radio receiver at given instabilities of the base generator.

An admissible error due to the instability of the base generator in equal to 0,1η, where η is the width of the discriminate characteristics of the tracking system. From this condition, for the given instability of the base generator $\Delta f_г$ will shall obtain a minimal admissible value of the equivalent band

$$f_ш=\frac{\Delta f_г f_г}{0{,}1}, \qquad (9).$$

where $f_г$ is the operating frequency of the base generator.

Thus, when calculating the probability and time characteristics of tracking failure (6), which are used to determine the effectiveness of the impact of noise on the receiver, the following procedure is used to acquire initial data:

a) the instabilities of the base generators, used in the device, are understood;
b) the equivalent bands PLL and DTS ($f_ш\varphi$ and $f_ш\tau$) are determined using the equation (9);

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c) in equation (6), the following is accepted

$$\Delta\varphi = \left(f_{ш}\varphi - \frac{\alpha}{2}\right)\left(1 - \frac{\tau_{э_0}}{\tau_э}\right)^2;\ \Delta\tau = \left(2\tau_э f_{ш}\tau - \beta\tau_э\right)\left(1 - \frac{\left|\tau_{p_0}\right|}{\tau_э}\right)\cos^2\varphi);$$

From equation (7) with the ratio $\frac{N_0}{A^2}$ << 1, which corresponds to the operation of a radio receiver during natural noise, we find the values N_φ and N_τ. To simplify the calculation, we can accept that $\alpha = f_{ш}\varphi$; $\beta = f_{ш}\tau$.

Solving elliptical differential equations (6), having substituted the numerical values of the parameters, which characterize the satellite signals, allows us to obtain the quantitative values of the average time and time dispersion to the moment of tracking failure in radio receivers of command and measuring systems in noise environments, having an intensity of N_0.

Since it has not been possible to solve this equation analytically [3, 8], the solution is sought for using numerical methods and the famous tridiagonal matrix algorithm [3].

In order to enable a high quality analysis of the influence of inverse modulation and cross coupling between PLL and DTS on the effectiveness of noise influence on the receiving device, the solution of the equation (6) is performed for the following variants:

- for PLL supposing that DTS has an ideal performance, i.e.

$$\tau_{p0} = 0, \frac{dt_k}{d\tau_{p_0}} = 0, \frac{d^2t_k}{d\tau_{p_0}^2} = 0, \frac{d^2t_k}{d\varphi_0 d\tau_{p_0}} = 0;$$

- for DTS supposing that PLL has an ideal performance, i.e.

$$\varphi_0 = 0, \frac{dt_k}{d\varphi_0} = 0, \frac{d^2t_k}{d\varphi^2} = 0, \frac{d^2t_k}{d\varphi_0 d\tau_{p_0}} = 0$$

- for joint performance of PLL and DTS.

When solving the equation, the values of the parameters of the radio signals are taken as $f_{г\varphi}$ = 1 500 MHz; $f_{г\tau}$ = 10 MHZ; $\tau_э$ = 0,1 μs, where $f_{г\varphi}$ is the frequency of the DTS base generator частота опорного генератора PLL. The frequency instability of the base generator is accepted as equal to 10^{-9}. The solution of the equation (6) was performed for two values of T_0: 10^{-3}s and $2\cdot10^{-2}$ s. The initial tracking errors for frequency and phase were supposed as equal to errors caused by the instability of base generators, i.e. $\varphi_0 = 0{,}1\pi$, $\tau_{p0} = 0{,}01$ μs.

To get the results of the value, N_τ and N_φ, were determined for d = 10^{-4}. The further decrease of *d* while finding the values of N_φ and N_τ did not significantly affect the results if the calculation; this allows us obtain guarantied assessment during investigations in conditions of greater ambiguity for possible real values of N_φ and N_τ.

During calculations, we have determined that the value of the average time to tracking failure corresponds to the mean square value of time deviation to tracking failure.

CONCLUSION

Based on the conducted analysis, we can conclude that during the evaluation of the noise immunity of a radio receiver operating in the tracking mode in noise environments, it is enough to study the tracking failure in the PLL at real initial shifts of signal frequency and phase and the ideal operating of the DTS.

Let us determine the threshold value of the noise to signal ratio $K_п$ for a noise interference which matches the signal spectrum. Considering that actual interfering signal can be replaced by a fictive flat noise with a correlative function.

$$K_{П}\left(t_2 - t_1\right) = \frac{N_0}{2}\delta\left(t_2 - t_1\right),$$

where $N_0 = \dfrac{2P_{П}}{\Delta f_{СП}}$; Pп is the noise level at output from the receiver.

In result we get

$K_п=(P_п/P_c)_{пор}=4\delta_П/\Delta f_{СП}$,

where $P_п / P_c$ is the ratio between the noise level to the signal at output.

Considering that in real receivers more than half of the signal power is lost during processing [9], we shall get

$$K_п = 2\delta_П/\Delta f_{СП}. \qquad (10).$$

The results of the calculations using the formula (10) and considering the calculated above threshold values of the ratio $\dfrac{N_0}{A^2 T_0}$ are given in the table.

Table 1. Threshold values of the noise to signal ratio for noise.

$f_{шф}$, Hz	6	10	25	40	50	70
$K_{пшт}$, dB	37	36	30	27	26	25

The realizable values of the minimal width of the equivalent band of the PLL system for radio receiving devices of command and measuring systems are estimated at 20 Hz. Consequently, the threshold ratio of noise to signal is respectively 42 dB and 32 dB.

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