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Homotopy Perturbation Method for SEIR Epidemic model with treatment.

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ABSTRACT

In this paper, we examine a disease transmission SEIR (susceptible, exposed, infectious, and recovered) model that includes treatment. Treatment is essential in battling overpowering illnesses. We consider r as the parameter that depicts the social energy to fight the infection. Generally, nonlinear problems have some difficulties, and their solutions are difficult to obtain. We investigate the logical arrangement of the model using the Homotopy perturbation technique to solve the nonlinear issue. Using this method, we solve the problem analytically. We also explain the numerical simulation and their results.

Keywords: Homotopy perturbation method, Epidemic SEIR models,Treatment function

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INTRODUCTION

When analyzing the spread and control of infectious diseases mathematical displaying turns like huge devices For examining the spread of infectious diseases, much effort has been directed into creating sensible numerical models [1-28].

Normally, epidemic modelling includes susceptible, infectious, and recovered populations. However, in the real world, before the hosts become infectious, many diseases have a period of incubation inside the hosts [1, 2]. If we include the incubation period of the hosts, the model becomes an SEIR (susceptible, exposed, infectious, and recovered) model [15]. Treatment plays a critical role in controlling or reducing various infectious diseases, including measles,and tuberculosis. [10, 11, 12, 17].

In our most recent work, we address the impact of cures on disease dynamics(13&16). In traditional pandemic models, the treatment rate is believed to be relative to the wide assortment of infective, while the limit of cure is not come to.Here, the treatment rate is thought to be relative to the quantity of infective persons, whereas the limit of treatment is not come to and takes the maximal limit [8, 14].
Consider the treatment function of a disease.

$$T(I)=\begin{cases} I & if\ 0<I\le I_0 \\ k & if\ I>I_0 \end{cases}$$

Where k is a positive constant and I is the number of infected individuals [3].

Most by far of the natural issues as disease models are intrinsically nonlinear. In this way it's troublesome as well as constantly difficult to find the exact solutions for these non-straight biological problems [5, 6].

Thus, researchers aim to discover numerical techniques or perturbation methods that exactly solve these non-linear problems [7, 8, 9]. Here, we apply a HPM approach to find analytical numerical solutions. This method yields solutions in convergent series forms with successfully process able terms [10, 11, 12, 13 , 14]. We found that this technique is advantageous and can be applied to wide range of scenarios. It should be noted that the methods presented in this paper can be stretched out for finding the expository arrangement of the nonlinear differential conditions [15, 16, 17].

MATERIAL AND METHODS

In the current SEIR model with treatment,(S(t), E(t), I(t), and R(t) represent the susceptible, exposed, infectious, and recovered populations respectively. At this point, we accept steady recruitment of individuals into the susceptible population by birth or migration [18, 19, 20]. The exposed individuals are those that have come into contact with at least one infectious person. After a latency period, the infective person is either immune or recovered.Treatment is included in all the epidemic models [21, 22, 23].

In this model we consider A is the recruitment rate

(i) βSI with β > zero, as an effective infection rate is the range of infected humans that increases and proportional to the wide variety of infectious
(ii) exposed class individuals will transfer to infectious class with a rate φ
(iii) the removal rate γ is the number of infectious individuals transfer to recovered section (iv) μ is taken as natural.

Demise charge and μ_t as disease associated death rate (v) r is considered as treatment rate which proportional to the number of infective individuals.

$$\frac{dx}{dt}=A-\beta xy-\mu x$$

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$$\frac{d\tilde{y}}{dt} = \beta xy - (\varphi + \mu)\tilde{y} \quad (1)$$

$$\frac{dy}{dt} = \varphi\tilde{y} - (\gamma + \mu + \mu_t + \mathrm{r})\,\mathrm{y}$$

$$\frac{dz}{dt} = \gamma y - \mu z + ry$$

HOMOTOPY PERTURBATION METHOD

Consider the non-linear differential equation (2) which can be solved by homotopy perturbation method [14,15]. $\mathrm{H(u)} = g(\mathrm{v}), \mathrm{v} \in \Omega$ with boundary condition

$$K = \left(u, \frac{\partial u}{\partial t}\right)0\ , \mathrm{v} \in \Gamma.$$

The operator H contains two parts H_1 and H_2.

$$H_1(u) + H_2(u) = g(v), v \in \Omega.$$

He constructed a homotopy which satisfies

$$M(w, p) = (1 - p)[\,H_1(w) - H_1(u_0)] + p[H(u) - g(v)] = 0,$$

Where $p \in [0,1]$ is an inserting factor, and u is an initial approximate value .we have

$$H_1(w, 0) = H_1(v) - H_1(u_0) = 0,\ M(w, 1) = H(u) - g(v) = 0.$$

Then by using HPM, the solution can be expressed as a series in p in the form

$$w = w_0 + pw_1 + p^2 w_2 + p^3 w_3 + \ldots + \infty.$$

When $p \to 1$, the approximate solution

$$u = \lim_{p \to 1} w = w_0 + w_1 + w_2 + w_3 + \ldots + \infty.$$

HOMOTOPY PERTURBATION METHOD TO A MODEL FOR SEIR MODEL

In this area, we apply the homotopy perturbation method to the above equation (1).

As indicated by homotopy perturbation [13-16], we determine

$$\frac{dx}{dt} - A + \beta xy + \mu x = 0$$

$$\frac{d\,y}{dt} - \beta xy + (\varphi + \mu)y = 0$$

$$\frac{dy}{dt} - \varphi y + (\gamma + r + \mu + \mu_t)\,\mathrm{y} = 0 \quad (2)$$

$$\frac{dz}{dt} - (\gamma + r)\,\mathrm{y} + \mu z = 0$$

To obtain the solution of the above system of equation we first calculate a homotopy

$$(1-\mathrm{p})\left[\frac{dx}{dt}-A+\mu x\right]+p\left[\frac{dx}{dt}-A+\beta xy+\mu x\right]=0$$

$$(1-\mathrm{p})\left[\frac{dy}{dt}+(\varphi+\mu)y\right]+p\left[\frac{dy}{dt}-\beta xy+(\varphi+\mu)y\right]=0$$

$$(1-\mathrm{p})\left[\frac{dy}{dt}+(\gamma+r+\mu+\mu_t)\,\mathrm{y}\right]+p\left[\begin{matrix}\frac{dy}{dt}-\varphi y\\ +(\gamma+r+\mu+\mu_t)\,\mathrm{y}\end{matrix}\right]=0 \qquad (3)$$

$$(1-\mathrm{p})\left[\frac{dz}{dt}+\mu z\right]+p\left[\frac{dz}{dt}-(\gamma+r)\,\mathrm{y}+\mu z\right]=0$$

Let we consider

$$x=x_0+px_1+p^2x_2+\ldots$$

$$y=y_0+py+p^2y+\ldots$$

$$y=y_0+py_1+p^2y_2+\ldots \qquad (4)$$

$$z=z_0+pz_1+p^2z_2+\ldots$$

Consider the coefficient of p^0 we get

$$\frac{dx_0}{dt}-A+\mu x_0=0$$

$$\frac{dy_0}{dt}+(\varphi+\mu)y_0=0$$

$$\frac{dy_0}{dt}+(\gamma+r+\mu+\mu_t)\,\mathrm{y}_0=0 \qquad (5)$$

$$\frac{dz_0}{dt}+\mu z_0=0$$

Consider the coefficient of p^1 we get

$$\frac{dx_1}{dt}+\mu x_1+\beta x_0y_0=0$$

$$\frac{dy_1}{dt}+(\varphi+\mu)y_1-\beta x_0y_0=0$$

(6)

$$\frac{dy_1}{dt}+(\gamma+\mathrm{r}+\mu+\mu_t)\,\mathrm{y}_1-\varphi y_0=0$$

$$\frac{dz_1}{dt}+\mu z_1-(\gamma+\mathrm{r})\,\mathrm{y}_0=0$$

Solution of x_0, y_0, y_0, z_0 (4) implies

$$x_0(\mathrm{t})=a_1\,\mathrm{e}^{-\mu t}+\frac{A}{\mu}$$

$$y_0(\mathrm{t})=a_2\,\mathrm{e}^{-(\mu+\varphi)\mathrm{t}}+\frac{1}{\mu+\varphi}$$

$$y_0(\mathrm{t})=a_3\,\mathrm{e}^{-at}+\frac{1}{a}$$

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$$z_0(t) = a_4 e^{-\mu t} + \frac{1}{\mu}$$

Where $a_1 = \left(i_1 - \frac{A}{\mu}\right)$, $a_2 = \left(i_2 - \frac{1}{\mu+\varphi}\right)$, $a_3 = \left(i_3 - \frac{1}{a}\right)$, $a_4 = \left(i_4 - \frac{1}{\mu}\right)$

(5) implies

$$x_1(t) = \frac{\beta a_1 a_3}{a} e^{-\mu t}\left[e^{-at} - 1\right] + \frac{\beta a_3 A}{\mu - a}\left[e^{-\mu t} - e^{-at}\right] + \frac{\beta A}{a\mu^2}\left[e^{-\mu t} - 1\right]$$

$$y_1(t) = \frac{\beta a_1 a_3}{\varphi - a} e^{-\mu t}\left[e^{-at} - e^{-\varphi t}\right] + \frac{\beta a_1}{\varphi a} e^{-\mu t}\left[1 - e^{-\varphi t}\right] + \frac{\beta A a_3}{\mu(\varphi + \mu - a)}\left[e^{-at} - e^{-(\varphi+\mu)t}\right]$$

$$+ \frac{\beta A}{a\mu(\varphi+\mu)}\left[1 - e^{-(\varphi+\mu)t}\right]$$

$$y_1(t) = a_2 \frac{\varphi}{a - (\varphi+\mu)}\left[e^{-(\varphi+\mu)t} - e^{-at}\right] + \frac{\gamma}{a(\varphi+\mu)}(1 - e^{-at})$$

$$z_1(t) = a_3 \frac{(\gamma + r)}{\mu - a}\left[e^{-at} - e^{-\mu t}\right] + \frac{(\gamma + r)}{a\mu}\left[1 - e^{-\mu t}\right]$$

The approximate analytical solutions of the above systems

$$x = x_0 + x_1, y = y_0 + y_1, y = y_0 + y_1, z = z_0 + z_1$$

$$x(t) = a_1 e^{-\mu t} + \frac{A}{\mu} + \frac{\beta a_1 a_3}{a} e^{-\mu t}\left[e^{-at} - 1\right] + \frac{\beta a_3 A}{\mu(\mu - a)}\left[e^{-\mu t} - e^{-at}\right] + \frac{\beta A}{a\mu^2}\left[e^{-\mu t} - 1\right]$$

$$\tilde{y}(t) = a_2 e^{-(\mu+\varphi)t} + \frac{1}{\mu+\varphi} + \frac{\beta a_1 a_3}{\varphi - a} e^{-\mu t}\left[e^{-at} - e^{-\varphi t}\right]$$

$$+ \frac{\beta a_1}{\varphi a} e^{-\mu t}\left[1 - e^{-\varphi t}\right] + \frac{\beta A a_3}{\mu(\varphi + \mu - a)}\left[e^{-at} - e^{-(\varphi+\mu)t}\right] + \frac{\beta A}{a\mu(\varphi+\mu)}\left[1 - e^{-(\varphi+\mu)t}\right]$$

$$y(t) = a_3 e^{-at} + \frac{1}{a} + a_2 \frac{\varphi}{a - (\varphi+\mu)}\left[e^{-(\varphi+\mu)t} - e^{-at}\right] + \frac{\gamma}{a(\varphi+\mu)}(1 - e^{-at})$$

$$z(t) = a_4 e^{-\mu t} + \frac{1}{\mu} + a_3 \frac{(\gamma + r)}{\mu - a}\left[e^{-at} - e^{-\mu t}\right] + \frac{(\gamma + r)}{a\mu}\left[1 - e^{-\mu t}\right]$$

RESULTS AND DISCUSSION

We numerically solve the SEIR model show by utilizing HPM and the outcomes as plots offered for justification purpose [24, 25, 26, 27, 28]. To determine the theoretical results gained in this paper, we will provide some numerical simulations.

For the following parameter values as

$S = 4;\ E = 1;\ I = 1;\ R = 1;\ A=3;\ \beta=0.5;\ \varepsilon=0.7;\ \mu=0.012;\ \mu_t=0.1,\ r=0.8;\ \gamma=0.5.$

The solution of the model is obtainable in the form of plots. Our analytical solution gives that for non-linear ODEs just a few iterations of HPM with good results. In our simulations, we suggested that epidemic treatment may indeed be an effective way of preventing an epidemic. It is important to also estimate the number of treatments required. In this numerical simulation, Fig.1 represents the analytical solutions of SEIR model by using HPM, Fig.2 represents the solutions is obtained from reducing the infection rate by 0.2 and also increase the treatment rate by 0.1, Fig.3 shows the solutions is obtained from reducing the infection rate by 0.1 and also increase the treatment rate by 0.1 and Fig.4 represents the solutions is obtained from reducing

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the infection rate by 0.1 and also increase the treatment rate by 0.1. From Figure 1 to 4, we conclude increasing the treatment rate(which results in decreasing the infection rate) reduces the infected population.

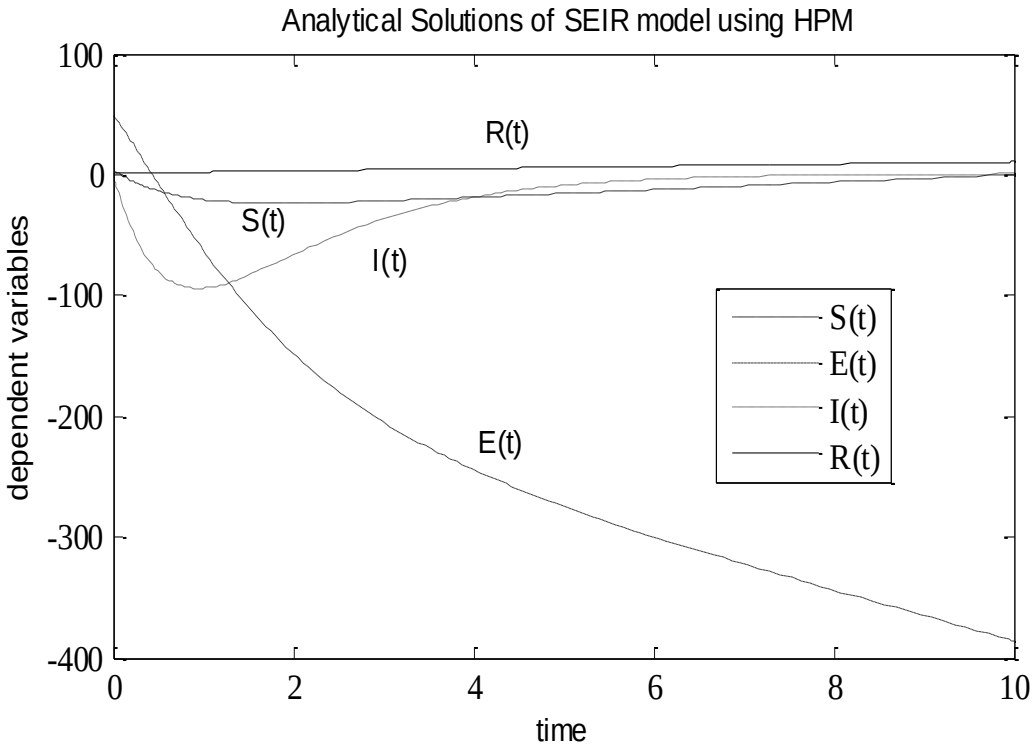

Figure-1 represents the analytical solutions of SEIR model by using HPM.

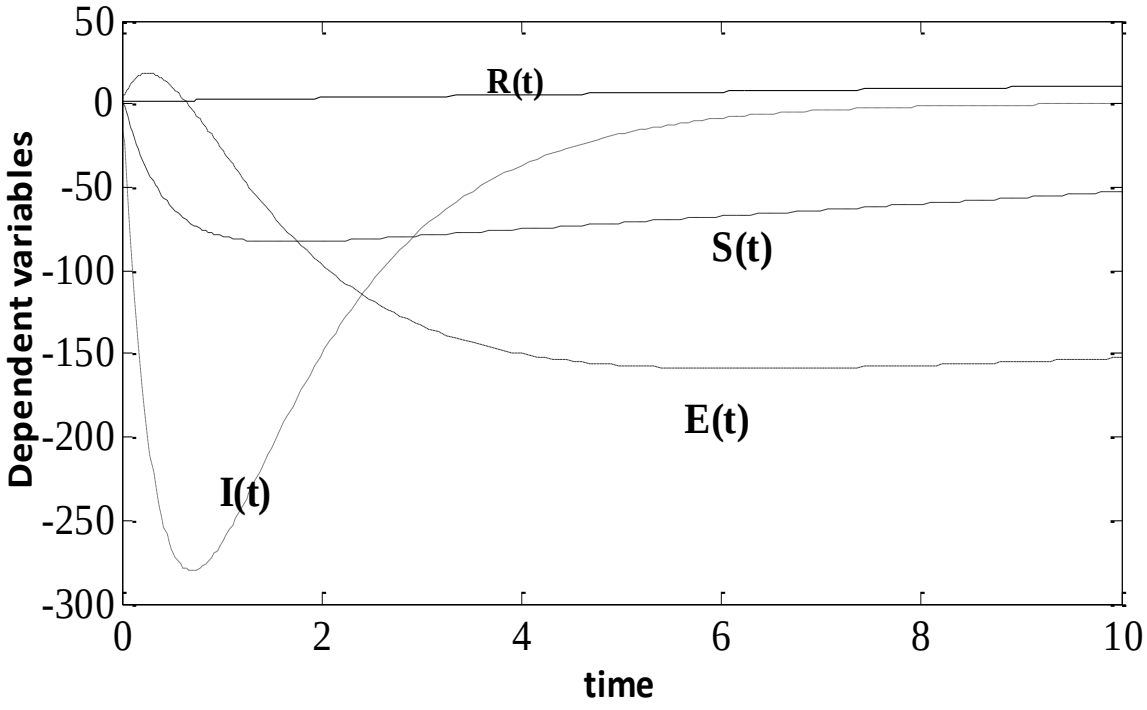

Figure-2 represents the solutions is obtained from reducing the infection rate by 0.2 and also increase the treatment rate by 0.1

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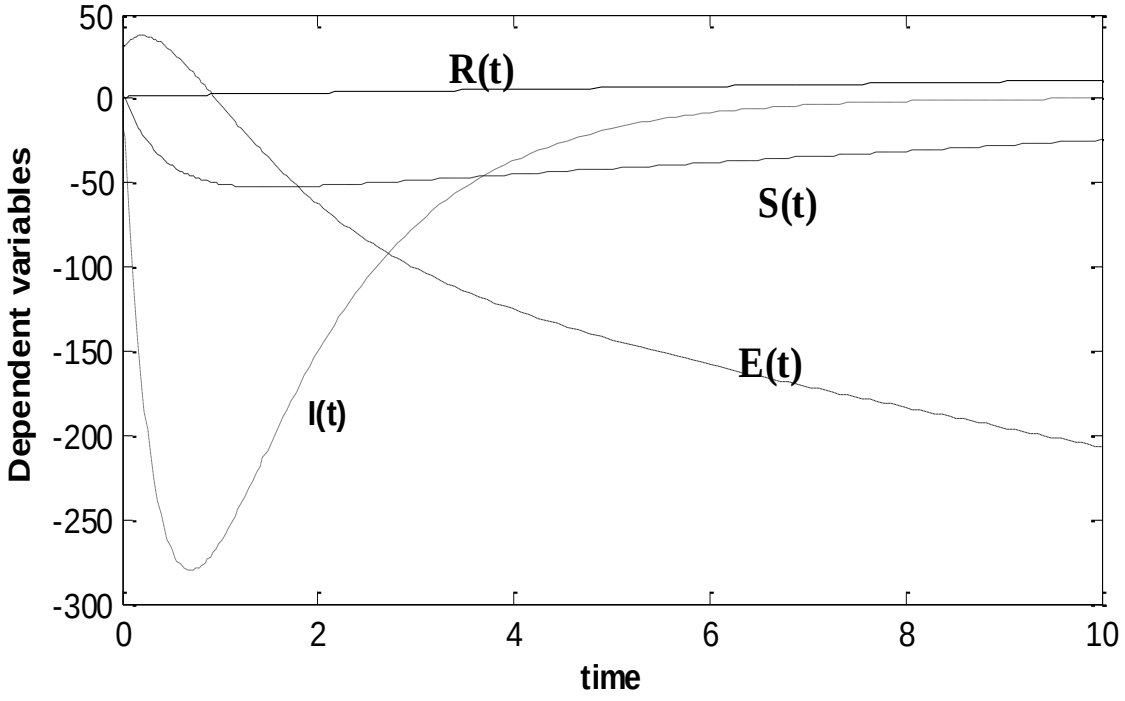

Figure-3 shows the solutions is obtained from reducing the infection rate by 0.1 and also increase the treatment rate by 0.1

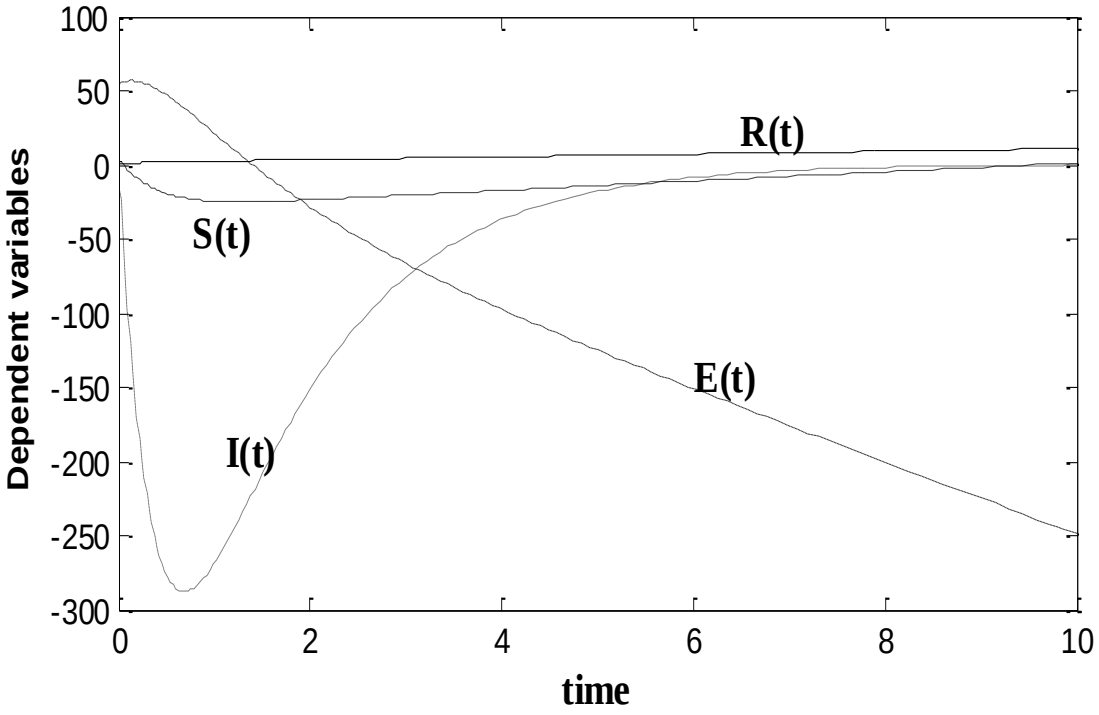

Figure-4 represents the solutions is obtained from reducing the infection rate by 0.1 and also increase the treatment rate by 0.1.

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CONCLUSIONS

In this paper, we used homotopy technique for finding the course of action of nonlinear differential condition structures of SEIR model. We favor the accuracy and feasibility of these techniques by strategies for settling differential condition systems. we attained the answer of zeroth and first order. It is simple and extremely gainful device for our system of non-linear ODEs just a few iterations of HPM give a good results. Methods to persisting with epidemic disease that have been proposed recently include treatment of susceptible individuals, treatment throughout an epidemic of exposed individuals recognized by contact tracing, and treatment during an epidemic reduces the number of infective individuals .

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REFERENCES

[1] Ariel.P.D, Alternative approaches to construction of Homotopy perturbation Algorithms,
[2] *Nonlinear Science Letters*,1, 43–52, (2010)
[3] Cai.L,Li.X, Ghosh.M and Guo .B, Stability analysis of an HIV/AIDS epidemic model with treatment, *Journal of computational and applied mathematics*, 229, 313-323. (2009)
[4] Ghori.Q.K,Ahmed.M, and Siddiqui, Application of homotopy perturbation method to squeezing flow of a newtonian fluid, *International Simulation*,. 8, 179–184, (2007)
[5] He J.H., A Coupling Method of A Homotopy Technique and A Perturbation technique for Non-Linear Problems*, International journal of Nonlinear Mechanics*,35,37-43(2000)
[6] He.J.H, Homotopy perturbation method: a new nonlinear analytical technique,*Applied Mathematics and Computation*, 135, 73–79, (2003)
[7] He.J.H,A simple perturbation approach to Blasius equation, *Applied Mathematics and Computation,* 140, 217–222,(2003)
[8] Kar.T.K and Ashim Batabyal Modeling and analysis of an epidemic mode with non-monotonic incidence rate under treatment*, Journal of mathematics research*, 2,103-115. (2010)
[9] Öziş.T and Yildirim.A, A comparative study of He's Homotopy Perturbation Method for determining frequency-amplitude relation of a nonlinear oscillator with discontinuities, *International Journal of Nonlinear Sciences and Numerical Simulation*,8, 243–248, (2007)
[10] Sarah A. and Al-Sheikh King, Modeling and Analysis of an SEIR Epidemic Model with a Limited Resource for Treatment, *Global Journal of Science Frontier Research Mathematics and Decision Sciences*, 12,57-66 (2012)
[11] Shu.H, Fan.D, and Wei.J, Global Stability of multi-group SEIR epidemic models with distributed delays and nonlinear transmission. Nonlinear Analysis, *Real World Applications*, 13, 1581-1592. (2012)
[12] Sun.C and Hsieh.Y,Global analysis of an SEIR model with varying population size and vaccination, *Applied mathematical modelling*, 34, 2685-2697. (2010).
[13] Wang.J, Liu.S, Zheng.B and Takeuchi.Y, Qualitative and bifurcatio analysis using an SIR model with a saturated treatment function.,*Mathematical and Computer modelling,* 55, 710-722. (2012)
[14] Wendi Wang ,Backward bifurcation of an epidemic model with treatment.,*Mathematical biosciences*, 201, 58-71(2006)
[15] Xueyong Zhou and Jingan Cui,Analysis of stability and bifurcation for an SEIR epidemic model with saturated recovery rate.,*Commun Nonlinear Sci Numer Simulat* ,16,4438-4450.(2011)
[16] Zhan.X,Liu .X,Backward bifurcation of an epidemic model with saturated treatment function. *Journal of mathematical analysis and applications*, 348, 433-443(2008)
[17] Zhou .X and Cui .J, Analysis of stability and bifurcation for an SEIR epidemic model with saturated recovery rate, *Communications in nonlinear science and numerical simulation*, 16, 4438-4450 (2011)
[18] Balamuralitharan S, D.Seethalakshmi, Numerical Simulations for Parameter Estimation Model of SIR with Replacement Numbers, *Global Journal of Pure and Applied Mathematics* (GJPAM), ISSN 0973-1768, Volume 12,Number 1 (Jan 2016), pp.118-123.
[19] Balamuralitharan S, S.Geethamalini, Within Host Virus Models with Sensitive and Resistant Strain in EIAV, *Global Journal of Pure and Applied Mathematics* (GJPAM), ISSN 0973-1768, Volume 12,Number 1 (Jan 2016), pp.329-334.

[20] S. Geethamalini and S. Balamuralitharan, Homotopy Perturbation Method for Solving A Model for EIAV Infection, *International Journal of Control Theory and Applications* (IJCTA), 9(28), 2016, pp. 439-446.
[21] S.Balamuralitharan and M.Radha, STABILITY ANALYSIS OF CHOLERA - CARRIER DEPENDENT INFECTIOUS DISEASE, *International Journal of Pure and Applied Mathematics*, Volume 113, No. 11, 2017, 234 - 242.
[22] S.Balamuralitharan and V.Geetha, ANALYTICAL APPROACH TO SOLVE THE MODEL FOR HIV INFECTION OF $CD4^{+}$T CELLS USING LADM, *International Journal of Pure and Applied Mathematics*, Volume 113, No. 11, 2017, 243 - 251.
[23] S.Balamuralitharan and S.Geethamalini, PARAMETER ESTIMATION OF MODEL FOR EIAV INFECTION USING HPM, *International Journal of Pure and Applied Mathematics*, Volume 113 No. 12, 2017, 196 - 204.
[24] G. Arul Joseph and S. Balamuralitharan, A MATHEMATICAL MODELING AND SIMULATION OF NON LINEAR ORDINARY DIFFERENTIAL EQUATIONS USING HPM, *ARPN Journal of Engineering and Applied Sciences*, VOL. 13, NO. 7, APRIL 2018, 2685- 2689.
[25] S. Balamuralitharan, M.Radha, BIFURCATION ANALYSIS IN SIR EPIDEMIC MODEL WITH TREATMENT, *IOP Conf. Series: Journal of Physics: Conf. Series* 1000 (2018) 012169 doi :10.1088/1742-6596/1000/1/012169
[26] V.Geetha And S. Balamuralitharan, STABILITY ANALYSIS OF HOST DYNAMICS FOR HIV, *IOP Conf. Series: Journal of Physics: Conf. Series* 1000 (2018) 012022 doi :10.1088/1742-6596/1000/1/012022
[27] S.Balamuralitharan and S.Geethamalini, Solutions Of The Epidemic Of EIAV Infection by HPM, *IOP Conf. Series: Journal of Physics: Conf. Series* 1000 (2018) 012023 doi :10.1088/1742-6596/1000/1/012023
[28] G. Arul Joseph and S.Balamuralitharan, A Nonlinear Differential Equation Model of Asthma Effect Of Environmental Pollution using LHAM, *IOP Conf. Series: Journal of Physics: Conf. Series* 1000 (2018) 012043 doi :10.1088/1742-6596/1000/1/012043
[29] Geethamalini S. and Balamuralitharan S, Equine infectious anaemia virus dynamics and stability analysis: The role of agar gel immunodiffusion test and enzyme immunoabsorbent assay, *Research Journal of Biotechnology,* Vol. 13 (5) May (2018), 28- 33.