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Developing Stochastic Linear Programming Model to Optimize Agricultural Production under Uncertain Flood Influence.

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ABSTRACT

Though, once, agriculture had a major contribution in our GDP, day by day its contribution is rapidly decreasing. The paper proposed a decision making model to maximize agricultural production under the uncertain environment dominated by flood. In Bangladesh flood plays a vital role negatively for agricultural production where all inputs (seeds, credit, fertilizer, irrigation) are considered here certain except land that is affected by flood. The proposed method uses stochastic liner programming model under flood uncertainty which can be determined by Markov chains. The model primarily intended to measure the probability of flood frequency.

Keywords: Stochastic, Stationary, Markov chain.

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INTRODUCTION

Climate change is one of the critical issue in the word and Bangladesh is most vulnerable among the affected countries. Geographically Bangladesh is low positioning flood plain country (BWDB, 2014). Therefor Bangladesh has been experienced different types of natural digester like flood, cyclone, tsunami etc. Each and every year our agricultural land mostly dominated by flood. And we have seen four types of flood happening in Bangladesh like flash floods, riverine floods, rain floods and storm surge floods (Mirza, 2002). Flash flood happened when sudden sharp rise of water level, Riverine floods from the slopping of major rivers and their branches and distributaries generally rise and fall slowly over 10–20 days or more and can cause wide damage to crops, Rain floods are caused by high intensity local rainfall of long duration in the monsoon, Storm surge floods which consists of large estuaries, extensive tidal flats, and low-lying islands (Mirza, 2002).

Bangladesh is a small but over populated country. Only 25 percent of its land are cultivable. Areas under two times, three times or four times cropping are showing an increasing movement over time. (BBS, 1999, 2012, 2015). There are several factors decreasing the cultivable land. Though in long run flood bring a benefit for the land through mineral away and enrich the soil, but in short run most of the time it destroy our crops production (L. Banerjee, 2010). In this paper, we consider the short run perspective of flood. Our objective is to measure probability which may affect the agricultural land. In Bangladesh from 1953 to 2014, flood flooded the land in different level (Annual flood report, 2014). Since flood is uncertain and it acts as a barrier to our production, we need to determine the probability of flood.Olsen et al. (2015) identified and compared three different methods for estimating the expected annual damage (EAD) based on unit costs of flooding of urban assets. There are two basic approaches for estimating flood impacts: the first approach employs unit loss models and the second employs models, which estimate the linkage effects, or inter-sectorial relationships, of floods within economy (Parker, 1992; Islam, 2000). Dutta et al. (2003) introduced an integrated model for flood loss estimation in a river basin, which has two major components: a physically based distributed hydrologic model and a grid-based distributed loss estimation model. There are very few research available on the measures of frequency of flood. A physical based flood frequency model is developed by Kurthe et al. (1997). Stedinger et al., (1993) developed a methods for estimation of the Probability Distribution Function (PDF) of flood discharges at ungagged sites include: (1) transfer of stream flow records from a nearby river basin using a drainage area scaling relationship, followed by fitting a PDF and (2) use of regional flood frequency methods such as the index-flood or regional regression methods. N. K. Goel et al. (2000) developed a derived flood frequency distribution (DFFD) model to measures the flood frequency.
It is easier to make decision of using cultivable land if the probability of flood is known. This paper has two segment basically. One part is, we have measured the probability using Markov Chain method of occurring strength of flood to affect the country. Another part is, considering this flood probability, we have maximized our agricultural production.

A Markov chain is a system of elements making transition from one state to another over time. The order of the chain indicates the number of time steps in the past influencing the probability distribution of the present state, which can be greater than one. M. M. Hossain and S. Anam (2012) used Markov Chain method to measures the probability of rainfall in Dhaka station. Thomas and Fiering (1962) first of all used a first order Markov chain model to generate stream flow data. Srikanthan McMahon (1985) used and recommended a first order Markov chain model to generate annual rainfall data. The Markov Chain method is more significant for calculation of the transition probability from one state to another state (Feller, 1968).In this paper we break down the floods occurring in the seven state namely, state-1 (0-10% flooded land), state-2 (10%-20% flooded land), state-3 (20%-30% flooded land), state-4 (30%-40% flooded land), state-5 (40%-50% flooded land), state-6 (50%-60% flooded land), and state-7 (60%-70% flooded land) and first order Markov chain is used to determine the probability of flood frequency.S. Anam et al. (2017) developed stochastic linear programming model to maximize agricultural production under inputs uncertainty with respect to time. Therefore in this paper we develop a stochastic linear model to maximize agricultural production of Bangladesh under flood uncertainty.

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METHODOLOGY

Markov Chains:

Markov Chains are stochastic processes evaluating transition probabilities between discrete states in the observed systems. The Markov chain of the first order means that one for which each next state depends only on immediately preceding one. The second or higher order of Markov chains indicates that the processes in which the next state depends on two or more preceding ones.

Let be stochastic process, possessing discrete states space S = 2, , *v* In general, for a given sequence of time points $t_1 < t_2 < ...... < t_{n-1} < t_n$ the conditional probabilities should be:

$$\Pr\{X(t_n) = i_n | X(t_1 = i_1, ..., X(t_{n-1}) = i_{n-1}\} = \Pr\{X(t_n) = i_n | X(t_{n-1}) = i_{n-1}\}$$

The conditional probabilities $\Pr\{X(t) = j | X(s) = i\} = P_{ij}(s,t)$ are called transition probabilities of order = - from state *T* to state for all indices $0 \le s < t,$ with $1 \le i, j \le k$. They are denoted as the transition matrix For states, the first order transition matrix has a size of and takes the form:

$$P = \begin{bmatrix} P_{(1,1)} & P_{(1,2)} & ..... & P_{(1,k)} \\ P_{(2,1)} & P_{(2,2)} & ..... & P_{(2,k)} \\ .... & .... & .... & .... \\ P_{(k,1)} & P_{(k,2)} & ..... & P_{(k,k)} \end{bmatrix}$$

The state probabilities at time t is determined from the relative frequencies of the k states. A second order transition probability matrix is shown as below:

$$P = \begin{bmatrix} P_{(1,1),1} & P_{(1,1),2} & .... & P_{(1,1),k} \\ P_{(1,2),1} & P_{(1,2),2} & .... & P_{(1,2),k} \\ .... & .... & .... & .... \\ P_{(1,k),1} & P_{(1,k),2} & .... & P_{(1,k),k} \\ P_{(2,1),1} & P_{(2,1),2} & .... & P_{(2,1),k} \\ .... & .... & .... & .... \\ P_{(k,k),1} & P_{(k,k),2} & .... & P_{(k,k),k} \end{bmatrix}$$

Stochastic Linear Programming Model:

Given is the following linear programming problem with random parameters in the constraints:

$$\begin{aligned} & Minimize\ \ C^T X \\ & Subjectto \\ & \qquad AX \approx b \\ & \qquad TX \approx h \\ & \qquad X \ge 0 \end{aligned}$$

Where the relational symbol $\approx$ denotes =, $\ge$, or $\le$.

Then assume that the real value of (T,h) is not known, i.e., it is not known which instance of the model occurs. Furthermore, assume that the uncertainty is expressed by a probability distribution, e.g., so-called scenarios:

$$\Pr\{(T,h)=(T^s,h^s)\}=P_s, \text{ s} = 1,2,....,S .$$

In addition to this, assume that the probability distribution is known, e.g., by data, or experts, and that a deterministic linear program is a degenerate case. By stochastic linear programming it is possible to decide on x here and now, without knowing the real value of (T,h), but only by knowing its probability distribution. This is done by interpreting $Tx \geq h$ as a goal constraint, which is to be specified more precisely.

Data Collection:

Convenient method for sampling has been used to collect primary data. Five districts are considered in each group and from each selected district Two Thana (Police Stations) are selected. Though the actual target population is farmers, the information is collected from Thana assistant agricultural officer. Most of the farmers are not habituated with numerical measurement when they are farming. A questionnaire has been prepared and sent to the agricultural officer and through that the information is collected.

The secondary data are collected from different reports such as 'Year Book of Agricultural Statistics Bangladesh' published by Bangladesh Bureau of Statistics, 'Annual flood report 2014' published by Bangladesh Water Development Board.

RESULTS AND DISCUSSIONS OFTHE STUDY

Estimate Flood Probability Using Markov Chain:

There are six (land, fertilizer, pesticides, loan, irrigation and seeds) types of input that are used to produce agricultural goods. Among them land are affected by the flood. So at first, we measure the probability of flood to be occurred. Either flood will occur or not, so outcomes are discrete and we use Markov chain method of first order to measure the flood affecting probability.

Table 1: Year-wise Flood Affected Area in Bangladesh

Year	Flood Affected Area		Year	Flood affected area		Year	Flood affected area	
	Sq-Km	%		Sq-Km	%		Sq-Km	%
1954	36,800	25	1976	28,300	19	1998	1,00,250	68
1955	50,500	34	1977	12,500	8	1999	32,000	22
1956	35,400	24	1978	10,800	7	2000	35,700	24
1960	28,400	19	1980	33,000	22	2001	4,000	2.8
1961	28,800	20	1982	3,140	2	2002	15,000	10
1962	37,200	25	1983	11,100	7.5	2003	21,500	14
1963	43,100	29	1984	28,200	19	2004	55,000	38
1964	31,000	21	1985	11,400	8	2005	17,850	12
1965	28,400	19	1986	6,600	4	2006	16,175	11
1966	33,400	23	1987	57,300	39	2007	62,300	42
1967	25,700	17	1988	89,970	61	2008	33,655	23
1968	37,200	25	1989	6,100	4	2009	28,593	19

1969	41,400	28	1990	3,500	2.4	2010	26,530	18
1970	42,400	29	1991	28,600	19	2011	29,800	12
1971	36,300	25	1992	2,000	1.4	2012	17,700	20
1972	20,800	14	1993	28,742	20	2013	15,650	10.6
1973	29,800	20	1994	419	0.2	2014	36,895	25
1974	52,600	36	1995	32,000	22			
1975	16,600	11	1996	35,800	24			

In this paper we break down the floods occurring in the seven state namely, state-1 (0-10% flooded land), state-2 (10%-20% flooded land), state-3 (20%-30% flooded land), state-4 (30%-40% flooded land), state-5 (40%-50% flooded land), state-6 (50%-60% flooded land), and state-7 (60%-70% flooded land). According to this criteria the flood data become;

Table 2: Converted the flooded land in different state

Year	Flood	Year	Flood	Year	Flood
1954	3	1974	4	1994	1
1955	4	1975	2	1995	3
1956	3	1976	2	1996	3
1957	1	1977	1	1997	1
1958	1	1978	1	1998	7
1959	1	1979	1	1999	3
1960	2	1980	3	2000	3
1961	2	1981	1	2001	1
1962	3	1982	1	2002	1
1963	3	1983	1	2003	2
1964	3	1984	2	2004	4
1965	2	1985	1	2005	2
1966	3	1986	1	2006	2
1967	2	1987	4	2007	5
1968	3	1988	7	2008	3
1969	3	1989	1	2009	2
1970	3	1990	1	2010	2
1971	3	1991	2	2011	2
1972	2	1992	1	2012	2
1973	2	1993	2	2013	2
				2014	3

Analysis of the event of flood and Its Dependence by Markov Chain

The explanatory variables are measured in different kinds of scale but they are categorized in dichotomous form considering long past behavior of this meteorological factors in Bangladesh [Basu, A.N., 1971]. Notational, all dependent and independent variables are as follows:

The independent variables are:

1 = Current year Flood

= 1, if $_1$=0-10% land flooded
= 2, if $_1$=10%-20% land flooded
= 3, if $_1$=20%-30% land flooded
= 4, if $_1$=30%-40% land flooded
= 5, if $_1$=40%-50% land flooded
= 6, if $_1$=50%-60% land flooded
= 7, if $_1$=60%-70% land flooded

$_2$ = Previous year flood
= 1, if $_1$=0-10% land flooded
= 2, if $_1$=10%-20% land flooded
= 3, if $_1$=20%-30% land flooded
= 4, if $_1$=30%-40% land flooded
= 5, if $_1$=40%-50% land flooded
= 6, if $_1$=50%-60% land flooded
= 7, if $_1$=60%-70% land flooded

Transition Counts and Transition Probabilities for Order One

The transition counts for the first order Markov model are obtained by considering this year's and previous year's flood status of Bangladesh where state-1 (0-10% flooded land), state-2 (10%-20% flooded land), state-3 (20%-30% flooded land), state-4 (30%-40% flooded land), state-5 (40%-50% flooded land), state-6 (50%-60% flooded land), and state-7 (60%-70% flooded land).Table-4 gives the maximum likelihood estimates of transition probabilities for a first order Markov chain obtained directly by using transition counts by the formula:

$$P_{ij}(z) = \frac{n_{ij}}{ni}; n_i = \sum_i n_{ij} \; and \; P_{ij} = P[x_t = j / x_{t-1} = i]$$

Table 3: Frequency distribution for first order transition counts

Previous Year state of Flood	Current Year's State of Flood							Total
	State-1	State-2	State-3	State-4	State-5	State-6	State-7	
State-1	9	5	2	1	0	0	1	18
State-2	4	8	4	2	1	0	0	19
State-3	4	4	7	1	0	0	0	16
State-4	0	2	1	0	0	0	1	4
State-5	0	0	1	0	0	0	0	1
State-6	0	0	0	0	0	0	0	0
State-7	1	0	1	0	0	0	0	2
Total	18	19	16	4	1	0	2	60

Table 4: The maximum likelihood estimates of transition probabilities for the first order model.

Previous Year state of Flood	Current Year's State of Flood						
	State-1	State-2	State-3	State-4	State-5	State-6	State-7
State-1	0.36364	0.311	0.2027	0.064	0.015	0	0.041
State-2	0.246	0.341	0.28	0.0689	0.022	0	0.038
State-3	0.288	0.315	0.287	0.067	0.013	0	0.03
State-4	0.3	0.273	0.34	0.07	0.026	0	0
State-5	0.25	0.25	0.437	0.0625	0	0	0

State-6	0	0	0	0	0	0	0
State-7	0.375	0.263	0.275	0.059	0	0	0.0275

From table-4 and table-5, we calculate the expected flooded land using the following formula;

$Let, percentage\ of\ flooded\ land = L_i\ and\ its\ corresponding\ probability\ P(L_i), then\ expected\ flooded\ land;$

Table 5: Expected probability

Percentage of Flooded land	Probability to be flooded
L_1 =(10% flooded)X(Total land)	P(L_1)
L_2 =(20% flooded)X(Total land)	P(L_2)
L_3 =(30% flooded)X(Total land)	P(L_3)
L_4 =(40% flooded)X(Total land)	P(L_4)
L_5 =(50% flooded)X(Total land)	P(L_5)
L_6 =(60% flooded)X(Total land)	P(L_6)
L_7 =(70% flooded)X(Total land)	P(L_7)

$Then\ expected\ flooded\ land;$

$$E(L) = \sum_{i}^{7} L_i P(L_i)$$

Developing Stochastic Linear Model:

$Let\ us\ consider:$

$Decision\ variables:$

x_j $: Metricton\ jth\ crop\ producing\ in\ a\ perticular\ year\ (j = 1,2,3.....9)$

x_1 $: x\ amount\ of\ paddy\ should\ be\ produced$

x_2 $: x\ amount\ of\ wheat\ should\ be\ produced$

x_3 $: x\ amount\ of\ potato\ should\ be\ produced$

x_4 $: x\ amount\ of\ Jute\ should\ be\ produced$

x_5 $: x\ amount\ of\ pulses\ should\ be\ produced$

x_6 $: x\ amount\ of\ oil\ seed\ should\ be\ produced$

x_7 $: x\ amount\ of\ spices\ should\ be\ produced$

x_8 $: x\ amount\ of\ sugar\ cane\ should\ be\ produced$

x_9 $: x\ amount\ of\ vegetable\ should\ be\ produced$

$Parameters:$

l_j :Required land ('000' acres) to produce one metric ton jth crop ($j = 1,2,3.....9$).

f_j :Required fertilizer (metric ton) to produce one metric ton jth crop ($j = 1,2,3.....9$).

p_j :Required pesticides (metric ton) to produce one metric ton jth crop ($j = 1,2..9$).

c_j :Required credit (million) to produce one metric ton jth crop ($j = 1,2,3.....9$).

s_j :Required seeds (metric ton) to produce one metric ton jth crop ($j = 1,2,3.....9$).

w_j :Required irrigated land ('000' acres) to produce one metric ton jth crop ($j = 1,2,..9$).

y_{it} : Total ith agricultural inputs available for production in t year.

TP :Total agricultural production.

$$Max. \qquad TP = \sum_{j=1}^{9} x_j \qquad (1)$$

Subject to :

$$\sum_{j=1}^{9} l_j x_j \le y_{1t} - E(L) \qquad total\ land \qquad (2)$$

$$\sum_{j=1}^{9} f_j x_j \le y_{2t} \qquad total\ fertilizer \qquad (3)$$

$$\sum_{j=1}^{9} p_j x_j \le y_{3t} \qquad total\ pesticides \qquad (4)$$

$$\sum_{j=1}^{9} c_j x_j \le y_{4t} \qquad total\ loan \qquad (5)$$

$$\sum_{j=1}^{9} s_j x_j \le y_{5t} \qquad total\ seeds \qquad (6)$$

$$\sum_{j=1}^{9} w_j x_j \le y_{6t} \qquad total\ irrigation \qquad (7)$$

$$x_j \ge 0, \quad j = 1,2,3...............9 total\ irrigation \qquad (7)$$

$$x_j \ge 0, \quad j = 1,2,3...............9$$

The parameter $l_j, f_j, p_j, c_j, s_j, w_j$ are considered asd a constant value and datermined through questionnaire methos as a primary data.

On the other hand the parameter y_{it} is consider also constant and collected from secondary source given in the table Table – 6 except land. In this model land is considered as a stochastic parameter. The land is domin ated by flood. Here we use Markov chains method to find the flood frequency.

Numerical Illustration:

There are three different ways to use land in agriculture in our country. Some lands are used once because each and every year these land are flooded, some are used twice, and some are used thrice in agriculture in a year. So net used regularly land equal the summation of lands being used once, one time of land that used twice, and two times of that used thrice. Rest of land consider one time that used twice and one times that used thrice are more likely to affect the flood. To measure the flooded land this year, we consider last year only 10 percent land are flooded, the below table shows the probability of current year flood occurrence.

Table 6: Expected land

Percentage of Flooded land	Probability to be flooded
L_1 =(10% flooded)X(Total land)=3715	0.36364
L_2 =(20% flooded)X(Total land)=7430	0.311
L_3 =(30% flooded)X(Total land)=11145	0.2027
L_4 =(40% flooded)X(Total land)=14860	0.064
L_5 =(50% flooded)X(Total land)=18575	0.015
L_6 =(60% flooded)X(Total land)=22290	0
L_7 =(70% flooded)X(Total land)=26005	0.041

Therefore the expected flooded land= 8216.6141 Thousand metric ton
In 2013 according to BBS total agricultural inputs are given below;

Table 7: Available inputs

Inputs	Available ('000' acres/ metric ton)	Inputs	Available ('000' acres/ metric ton)
Land	37150-8216.6141=28933.3859	Loan	146670 (000, million)
Fertilizer	1259	Irrigation	17606
Pesticides	12.49	Seeds	118503

From primary data analysis we got the values of the coefficient $l_j, f_j, p_j, c_j, s_j, w_j$ that are given below;

Table 8: Coefficient of constraints

	Paddy	Wheat	Jute	Potato	Spice	Pulse	Sugarcane	Oilseeds	Vegetable
Land	0.38	0.705	1.235	0.1	1.4525	1.3725	0.031	1.235	0.21
Fertilizer	0.0675	0.1365	0.25	0.04	0.1525	0.24	0.0133	0.2	0.314
Pesticide	0.0005	0.0015	0.0026	0.0002	0.00025	0.0005	0.000055	0.0027	0.000475
Loan	1.25	1.875	2.5	1.25	2.5	1.25	1.25	2.5	1.25
Irrigation	0.38	0.705	1.235	0.1	1.4525	1.3725	0.031	1.235	0.21
Seeds	0.0055	0.03575	0.005	0.08	0.006	0.01675	0.375	0.005	0.000575

Insert the above values in our proposed stochastic linear model become;

$$Max. \qquad TP = \sum_{j=1}^{9} x_j$$

$$Subjectto:$$

$$L: 0.38x_1 + 0.705x_2 + 1.235x_3 + 0.1x_4 + 1.4525x_5 + 1.3725x_6 + 0.031x_7 + 1.235x_8 + 0.21x_9 = 28933.38$$

$$F: 0.0675x_1 + 0.1365x_2 + 0.25x_3 + 0.04x_4 + 0.1525x_5 + 0.24x_6 + 0.0133x_7 + 0.2x_8 + 0.314x_9 \geq 1259$$

$$P: 0.0005x_1 + 0.0015x_2 + 0.0026x_3 + 0.0002x_4 + 0.00025x_5 + 0.0005x_6 + 0.000055x_7 + 0.0027x_8 + 0.000475x_9 \geq 12.49$$

$$Lo: 1.25x_1 + 1.875x_2 + 2.5x_3 + 1.25x_4 + 2.5x_5 + 1.25x_6 + 1.25x_7 + 2.5x_8 + 1.25x_9 \geq 146670$$

$$I: 0.38x_1 + 0.705x_2 + 1.235x_3 + 0.1x_4 + 1.4525x_5 + 1.3725x_6 + 0.031x_7 + 1.235x_8 + 0.21x_9 \geq 17606$$

$$S: 0.0055x_1 + 0.03575x_2 + 0.005x_3 + 0.08x_4 + 0.006x_5 + 0.01675x_6 + 0.375x_7 + 0.005x_8 + 0.000575x_9 \geq 118503$$

Some

Additional constraints:

$$x_1 \geq 34710, x_2 \geq 1348, x_3 \geq 7501, x_4 \leq 9254, x_5 \geq 2409, x_6 \geq 726, x_7 \leq 4434, x_8 \geq 301, x_9 \leq 3729$$

$$x_j \geq 0, \quad j = 1,2,3..............9$$

Solution:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
34710	1348	7501	5248.085	2409	726	4434	301	0
Total Production	56677.085							

Use of Inputs:

Inputs	Used	Available
Land	28933.38	28933.38
Fertilizer	5272.885	1259
Pesticide	41.95104	12.49
Loan	84452.61	146670
Irrigation	28933.38	17606
Seeds	2387.317	118503

The above solution shows that land is properly used and total production 56677.085 thousand metric ton but other inputs are over used. Therefore to meet the additional inputs we have to import from abroad.

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