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# Calculation of Optimal Alternatives at Reconstruction of Residential Housing. 

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## ABSTRACT

Choosing the reconstruction method for improvement of built up areas depends on various factor, which served as a ground for their development (city planning, architecture and construction, economic, investment etc.), including attractiveness for investors and economic safety. One of the methods carry out mathematical calculations is the analytic hierarchy process, which requires creation of various alternatives of reconstruction projects. This article sets forth the method for choosing the optimal reconstruction alternative through comparison of various planning solutions. The proposed methodology makes it possible to eliminate the influence of possible subjectivity of expert judgments and withdraw from the expert method of optimality appraisal of various options of existing municipal development reconstruction.
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## INTRODUCTION AND GOAL SETTING

Reconstruction of residential housing requires a complex approach taking into account the improvement of space planning decisions of existing buildings or improving the efficient use of territories by means of demolition of obsolete and worn out houses and replacing them with modern buildings having more floors, the possibility of deconsolidation or densification of residential development, block settlement gardening and provision of all types of cultural domestic services to the population.

Carrying out of large-scale measures for reconstruction, especially in case housebreaking is necessary, must be technically justified and economically feasible [1].

The character of decisions related to choosing of a reconstruction method can be different. Such decisions depend on factors which form the basis of development thereof (town-planning, architectural and building, economic, investment, etc.), including investment attractiveness and economic reliability. This effect can be reached due to obtaining of housing accommodation with increased comfort and high operating performance within the reconstructed territory.

The optimal estimation of an existing residential development option implies the use of linear algebra methods. For the purpose of calculations hierarchy analysis technique, HAT, can be used, which was discovered by the American scientist Saaty [2].

## DEVELOPMENT OF MATHEMATICAL MODEL

The essence of HAT is as follows. There is a goal and a complex of simultaneously implemented current tasks which provide for achievement of this goal. The said goal is divided into a number of sub-goals or criteria (conditions), accomplishment of which ensures the fulfillment of the set task. Chosen criteria are compared with each other in couples (every criterion is compared with every other one), and the relative degree of importance of each criterion in a couple is determined according to 9 points system. Whole numbers are interpreted as follows: 1 - equal importance; 3 - weak advantage; 5 - strong advantage; 7 - very strong advantage; 9 - total supremacy; $2,4,6$, and 8 - borderline cases. The comparison procedure can be simplified by means of direct assignment of an absolute importance value with regard to each comparison element. In this case the easiest method that can be used is the 5 points system known to everyone from his or her schooldays: 1, 2, 3, 4, 5. For those who wish to keep 9 points system additional grading can be introduced: 1.5; $2.5 ; 3.5$; and 4.5. The following matrix is formed (Larichev OI, 2000):

$$
\left[\begin{array}{lllll}
a_{11} & a_{12} & a_{13} & \ldots \ldots & a_{1 n} \\
a_{21} & a_{22} & a_{23} & \ldots \ldots & a_{2 n} \\
a_{31} & a_{32} & a_{33} \ldots \ldots . & a_{3 n} \\
a_{n 1} & a_{n 2} & a_{n 3} \ldots \ldots . & a_{n n}
\end{array}\right]
$$

This matrix is antisymmetric, i.e. the following equation is valid: $a_{i j}=1 / a_{j i}$, where $i$ and $j$ indexes represent the number of a row and a column at the intersection of which an element is located. $a_{i j}$ coefficients can be selected from the fixed Saaty rating scale: $\{1 / 9 ; 1 / 8 ; 1 / 7 ; 1 / 6 ; 1 / 5 ; 1 / 4 ; 1 / 3 ; 1 / 2 ; 1 ; 2 ; 3 ; 4 ; 5 ; 6 ; 7 ; 8 ; 9\}$. Therefore, the general form of the paired judgment matrix is as follows [3]:


Solution of the task for determination of weighting coefficients for the following parameter groups: Residential areas, Non-residential areas, Availability of cultural-domestic servicing facilities, Reconstruction coefficients related to residential facilities and those taking non-residential facilities into account and the Cost
of construction. This determination will be carried out based on nonnegative parameter values, while with the increase of a parameter value the efficiency of the design solution is considered to be better.

## TASK SETTING AND PROBLEM SOLUTION METHODS

For instance, the following is is given: general objective (or objectives) of the problem solution; alternatives evaluation criteria; alternatives

It is required: to choose the optimal version of residential development reconstruction - an alternative.

The approach consists of a complex of stages.

1. At the first stage the task is represented in the form of a hierarchic structure having several levels: objectives - criteria - alternatives.
2. At the second stage the comparison results are translated into numbers.
3. Importance coefficients are calculated for elements of each level. At the same time the consistency of decision-maker's judgments is made.
4. Quantitative quality indicators are calculated for each of the alternatives, and the best option is determined.

It is worth noting that in the majority of cases the accuracy of methods with regard to approaches can be put in the following order of descending accuracy (ascending of standard deviation): 1. without housebreaking, superstructure, inserts between houses; 2. partial demolition and modernization of residential development; 3 . complete housebreaking and new construction.

Let us assume that the restructuring electoral commission preliminarily selected three options out of several possibilities: A, B and C (the parameters are chosen with regard to existing residential development). Then the structure of the solved task can be presented as shown in the figure 1.


Figure 1: Hierarchic scheme of choice of residential development

For the sake of simplicity the parameters are classified by four groups, while each group contains parameters which characterize one specific direction.

The following criteria (conditions) providing for achievement of this goal can be defined:
Residential construction

1. Total residential construction area
2. Residential accommodation
3. Population

Demolition
4. Total area of apartments
5. Total area of integrated non-residential facilities and amenity spaces
6. "Initial" construction

Availability of cultural-domestic servicing facilities
7. Health resorts for children: required / availability, $\%$
8. Schools: required / availability, \%
9. Garages ( $\mathrm{m} / \mathrm{m}$ ): required / availability, \%

## Resettlement coefficients

10. By residential facilities
reconstruction Cost of
11. The cost of square meters, landscaping and Communications, thousand rubles

Paired comparisons can be written in matrix form. The obtained matrix is in the form of a square array.
The dimension of the matrix will be $11 \times 11$ (since there are eleven specific parameters characterizing the development of territories). Therefore, it is necessary to complete 121 fields, eleven of which are already determined, namely, those that are on the main diagonal and are equal to one. 55 comparisons are made for 110 numbers remaining after completion of the main diagonal on Saaty's 9 point scale of relative importance; since other 55 comparisons are opposite, their estimates must be opposite values to previous estimated (see table 1 - to simplify the calculations, three parameters only are taken).

Ranks are assigned to each criterion and decision (approach) in Table 1.

For parameters the large values of which are of negative nature inverse scales were used. As a result, a matrix of standardized data expressed in coefficient form is created.

In the general case the matrix appears as follows:
The next step is to calculate the priority vector in accordance with this matrix.
Table 1 - Comparison elements and criteria

| Comparison <br> elements | $f_{1}$ | $f_{2}$ | $f_{3}$ |
| :--- | :---: | :---: | :---: |
|  | $a_{0}^{1}$ | $a_{0}^{2}$ | $a_{0}^{3}$ |
| Criteria | $a_{1}^{1}$ | $a_{1}^{2}$ | $a_{1}^{3}$ |
| Approach $d_{1}$ | $a_{2}^{1}$ | $a_{2}^{2}$ | $a_{2}^{3}$ |
| Approach $d_{2}$ |  |  |  |


| Approach $d_{3}$ | $a_{3}^{1}$ | $a_{3}^{2}$ | $a_{3}^{3}$ |
| :--- | :--- | :--- | :--- |

Note: a - grades (ranks), indexes (from this point onward): upper - number of the criterion, lower number of approach (o - for any approach).

Basic formulas are given in the following tables (in tables 2-6 three parameters are taken):

Further algorithm corresponds to a classical hierarchy analysis technique.
The following table 2 is completed for criteria, and importance coefficients are calculated as shown therein [4]:

The next step is to calculate the main eigenvector which after normalization becomes the priority vector. This vector is estimated in four following ways which are mentioned below in increasing accuracy of estimate.

Table 2 - Determining significance coefficient of the criteria

| Cri- <br> teria | $f_{1}$ | $f_{2}$ | $f_{3}$ | Weight | Importance <br> degree |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $f_{1}$ | 1 | $A_{0}^{12}=\frac{a_{0}^{1}}{a_{0}^{2}}$ | $A_{0}^{13}=\frac{a_{0}^{1}}{a_{0}^{3}}$ | $B_{0}^{1}=\sqrt[3]{1 \cdot A_{0}^{12} \cdot A_{0}^{13}}$ | $C_{0}^{1}=\frac{B_{0}^{1}}{B_{0}}$ |
| $f_{2}$ | $A_{0}^{21}=\frac{a_{0}^{2}}{a_{0}^{1}}$ | 1 | $A_{0}^{23}=\frac{a_{0}^{2}}{a_{0}^{3}}$ | $B_{0}^{2}=\sqrt[3]{A_{0}^{21} \cdot 1 \cdot A_{0}^{23}}$ | $C_{0}^{2}=\frac{B_{0}^{2}}{B_{0}}$ |
| $f_{3}$ | $A_{0}^{31}=\frac{a_{0}^{3}}{a_{0}^{1}}$ | $A_{0}^{32}=\frac{a_{0}^{3}}{a_{0}^{2}}$ | 1 | $B_{0}^{3}=\sqrt[3]{A_{0}^{31} \cdot A_{0}^{32} \cdot 1}$ | $C_{0}^{3}=\frac{B_{0}^{3}}{B_{0}}$ |
| $\Sigma$ |  |  |  | $B_{0}=B_{0}^{1}+B_{0}^{2}+B_{0}^{3}$ | 1 |

The next step is to calculate the main eigenvector which after normalization becomes the priority vector. This vector is estimated in four following ways which are mentioned below in increasing accuracy of estimate.

1. Sum up elements of each row and normalize by division of each sum by the sum of all elements. The sum of obtained results will be equal to one.

The first element of the resulting vector will be the priority of the first object; the second vector will be the priority of the second object, etc.
2. Sum up elements of each column and obtain reciprocal values of these sums. Normalize them in such a way that their sum is equal to one, and divide each reciprocal value by the sum of all reciprocal values.
3. Divide the elements of each column by the sum of elements of this column (i.e. normalize the column), then sum the elements of each obtained row and divide this sum by the number of elements in a row. This is the process of averaging over normalized columns.
4. In order to determine the relative value of each element it is necessary to find out the geometric average, and with this purpose multiply $n$ elements of each row and then find the $n$th root of the obtained result. Normalize the obtained numbers.

On the basis of the obtained comparison matrix a degree of importance of each criterion for achievement of the task in general is determined [5].

Approach matrices are completed for each criterion (tables 3).

Similarly, by means of pair-wise comparison, for each criterion matrices of methods are formed on the basis of which a degree of conformity of each method to each criterion is determined. From this point on, with due account for the degree of importance of each criterion a contribution (weighting factor) of each method for achievement of the target goal is determined.

Table 3 - Approach matrices are completed for each criterion

| Approac hes | $d_{1}$ | $d_{2}$ | $d_{3}$ | Weight | Degree of conformity to criterion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Criterion 1 |  |  |  |  |  |
| $d_{1}$ | 1 | $A_{12}^{1}=\frac{a_{1}^{1}}{a_{2}^{1}}$ | $A_{13}^{1}=\frac{a_{1}^{1}}{a_{3}^{1}}$ | $B_{1}^{1}=\sqrt[3]{1 \cdot A_{12}^{1} \cdot A_{13}^{1}}$ | $C_{1}^{1}=\frac{B_{1}^{1}}{B_{1}}$ |
| $d_{2}$ | $A_{21}^{1}=\frac{a_{2}^{1}}{a_{1}^{1}}$ | 1 | $A_{23}^{1}=\frac{a_{2}^{1}}{a_{3}^{1}}$ | $B_{2}^{1}=\sqrt[3]{A_{21}^{1} \cdot 1 \cdot A_{23}^{1}}$ | $C_{2}^{1}=\frac{B_{2}^{1}}{B_{1}}$ |
| $d_{3}$ | $A_{31}^{1}=\frac{a_{3}^{1}}{a_{1}^{1}}$ | $A_{32}^{1}=\frac{a_{3}^{1}}{a_{2}^{1}}$ | 1 | $B_{3}^{1}=\sqrt[3]{A_{31}^{1} \cdot A_{32}^{1} \cdot 1}$ | $C_{3}^{1}=\frac{B_{3}^{1}}{B_{1}}$ |
| $\Sigma$ |  |  |  | $B_{1}=B_{1}^{1}+B_{2}^{2}+B_{3}^{3}$ | 1 |
| Criterion 2 |  |  |  |  |  |
| $d_{1}$ | 1 | $A_{12}^{2}=\frac{a_{1}^{2}}{a_{2}^{2}}$ | $A_{13}^{2}=\frac{a_{1}^{2}}{a_{3}^{2}}$ | $B_{1}^{2}=\sqrt[3]{1 \cdot A_{12}^{2} \cdot A_{13}^{2}}$ | $C_{1}^{2}=\frac{B_{1}^{2}}{B_{2}}$ |
| $d_{2}$ | $A_{21}^{2}=\frac{a_{2}^{2}}{a_{1}^{2}}$ | 1 | $A_{23}^{2}=\frac{a_{2}^{2}}{a_{3}^{2}}$ | $B_{2}^{2}=\sqrt[3]{A_{21}^{2} \cdot 1 \cdot A_{23}^{2}}$ | $C_{2}^{2}=\frac{B_{2}^{2}}{B_{2}}$ |
| $d_{3}$ | $A_{31}^{2}=\frac{a_{3}^{2}}{a_{1}^{2}}$ | $A_{32}^{2}=\frac{a_{3}^{2}}{a_{2}^{2}}$ | 1 | $B_{3}^{1}=\sqrt[3]{A_{31}^{2} \cdot A_{32}^{2} \cdot 1}$ | $C_{3}^{2}=\frac{B_{3}^{2}}{B_{2}}$ |
| $\Sigma$ |  |  |  | $B_{2}=B_{1}^{1}+B_{2}^{2}+B_{3}^{3}$ | 1 |
| Criterion 3 |  |  |  |  |  |
| $d_{1}$ | 1 | $A_{12}^{3}=\frac{a_{1}^{3}}{a_{2}^{3}}$ | $A_{13}^{3}=\frac{a_{1}^{3}}{a_{3}^{2}}$ | $B_{1}^{3}=\sqrt[3]{1 \cdot A_{12}^{3} \cdot A_{13}^{3}}$ | $C_{1}^{3}=\frac{B_{1}^{3}}{B_{3}}$ |
| $d_{2}$ | $A_{21}^{3}=\frac{a_{2}^{3}}{a_{1}^{3}}$ | 1 | $A_{23}^{3}=\frac{a_{2}^{3}}{a_{3}^{3}}$ | $B_{2}^{2}=\sqrt[3]{A_{21}^{3} \cdot 1 \cdot A_{23}^{3}}$ | $C_{2}^{3}=\frac{B_{2}^{3}}{B_{3}}$ |
| $d_{3}$ | $A_{31}^{3}=\frac{a_{3}^{3}}{a_{1}^{3}}$ | $A_{32}^{3}=\frac{a_{3}^{3}}{a_{2}^{3}}$ | 1 | $B_{3}^{3}=\sqrt[3]{A_{31}^{3} \cdot A_{32}^{3} \cdot 1}$ | $C_{3}^{3}=\frac{B_{3}^{3}}{B_{3}}$ |
| $\Sigma$ |  |  |  | $B_{3}=B_{1}^{1}+B_{2}^{2}+B_{3}^{3}$ | 1 |

Then the final matrix is completed, and weighting factors for each approach are calculated as shown in table 4.

Table 4 - Calculation formula of weight factors

| Parameters | Criteria |  |  | Weighting factor for approach |
| :---: | :---: | :---: | :---: | :---: |
|  | $f_{1}$ | $f_{2}$ | $f_{3}$ |  |
| Degree of conformity <br> to criteria $d_{1}$ | $C_{1}^{1}$ | $C_{1}^{2}$ | $C_{1}^{3}$ | $k_{1}=C_{1}^{1} \cdot C_{0}^{1}+C_{1}^{2} \cdot C_{0}^{2}+C_{1}^{3} \cdot C_{0}^{3}$ |
| Degree of conformity <br> to criteria $d_{2}$ | $C_{2}^{1}$ | $C_{3}^{2}$ | $C_{3}^{3}$ | $k_{2}=C_{2}^{1} \cdot C_{0}^{1}+C_{2}^{2} \cdot C_{0}^{2}+C_{2}^{3} \cdot C_{0}^{3}$ |
| Degree of conformity <br> to criteria $d_{3}$ | $C_{3}^{1}$ | $C_{3}^{2}$ | $C_{3}^{3}$ | $k_{3}=C_{3}^{1} \cdot C_{0}^{1}+C_{3}^{2} \cdot C_{0}^{2}+C_{3}^{3} \cdot C_{0}^{3}$ |
| Degree of criteria <br> importance | $C_{0}^{1}$ | $C_{0}^{2}$ | $C_{0}^{3}$ | 1 |

Obtained values of weighting factors ( $k_{1}, k_{2}$ and $k_{3}$ ) are used in the course of choosing of the optimal version (the biggest value is the best solution) of the residential development reconstruction.

Methods of calculation of the value of standard deviation for each approach require separate consideration which is not given herein.

Therefore, the suggestion to directly assign grades reflecting the absolute importance of each of compared elements makes it possible, in conditions where these elements are scarce, to maintain the habitual for an engineer analytical approach to choosing of importance of calculation results without sacrificing the formalized algorithm which is inherent to hierarchy analysis technique.

After the hierarchy is built and the pair-wise comparison tables are compiled there is a stage at which hierarchic decomposition and relative values of weighting factors of the parameters in question are combined with the purpose of obtaining of a sensible solution to a multi-objective decision-making problem (table 5).

A set of local criteria is formed from pair-wise comparison groups. These criteria reflect the relative influence of elements to an element located one level above [6].

Table 5 - Relative influence of one element on the other, located on the upper level

|  |  |  |  |  |  |  |  | Schools: required / availability,\% |  |  | $\begin{aligned} & \dot{\varepsilon} \\ & \dot{\Delta} \\ & \dot{\Delta} \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total area of residential development | 1 | 6 | 3 | 3 | 2 | 3 | 3 | 3 | 3 | 1 | 1 | $\begin{aligned} & 8748 \\ & / 2.28 \end{aligned}$ | 0.21 |
| Housing unit density | 0.17 | 1 | 2 | 2 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | $\begin{gathered} \hline 1.36 \\ / 1.028 \\ \hline \end{gathered}$ | 0.09 |
| Population | 0.33 | 0.5 | 1 | 6 | 3 | 2 | 5 | 5 | 5 | 4 | 2 | $\begin{aligned} & 6000 \\ & / 2.2 \end{aligned}$ | 0.2 |
| Total area of apartments | 0.33 | 0.5 | 0.17 | 1 | 1.5 | 2 | 3 | 3 | 3 | 3 | 1 | $\begin{aligned} & \hline 6.82 \\ & / 1.19 \\ & \hline \end{aligned}$ | 0.11 |
| Total area of | 0.5 | 1 | 0.33 | 0.66 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.11 | 0.076 |


| embedded <br> non- <br> residential <br> and social <br> facilities |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Initial" <br> construction | 0.33 | 0.5 | 0.5 | 0.5 | 0 | 1 | 0 | 0 | 0 | 6 | 3 | 0.77 <br> $/ 0.09$ | 0.008 |
| Health resorts <br> for children | 0.33 | 0.5 | 0.2 | 0.33 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0.1 <br> $/ 0.81$ | 0.075 |
| School | 0.33 | 0 | 0.2 | 0.33 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.02 <br> $/ 0.71$ | 0.065 |
| Garages | 0.33 | 0 | 0.52 | 0.33 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0.17 <br> $/ 0.85$ | 0.076 |
| Resettlement <br> coefficients | 1 | 0 | 0.25 | 0.33 | 0 | 0.1 | 0 | 0 | 0 | 1 | 0 | 0.08 <br> $/ 0.79$ | 0.072 |
| Cost | 1 | 1 | 0.5 | 1 | 0 | 0.3 | 0 | 0 | 0 | 0 | 1 | 0.17 | 0.083 |
| Amount |  |  |  |  |  |  |  |  |  |  |  | 10.85 |  |

With the use of Microsoft Excel the main eigenvector of the matrix of pair-wise comparison of specific parameters that characterize the stable development of rural regions was found, and then it was normalized. As can be seen from the table, the priority vector was ( $0.21 ; 0.09 ; 0.2 ; 0.11 ; 0.076 ; 0.008 ; 0.075 ; 0.065 ; 0.076 ;$ $0.072 ; 0.083)$. Components of this vector are weighting factor for specific parameters that characterize the development of the territories.

## CONCLUSIONS

The suggested multiple criteria analysis system makes it possible to withdraw from the expert method of optimality appraisal of various options of existing municipal development reconstruction. The proposed methodology makes it possible to eliminate the influence of possible subjectivity of expert judgments.

The developed methodology allows us, due to mathematical analysis means and use of solely quantitative characteristics, to choose an optimal way of reconstructing the existing municipal development by means of comparison of presented design solutions with each other or in the course of comparison of design solutions with performance indicators of the existing planning.

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