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A Study of Various Fuzzy C Means Clustering Techniques for Segmentation.

Suganya SD*, Rajasekhar Reddy M, and Vaithiyanathan V.

School Of Computing, Sastra University, Tamil Nadu, India.

ABSTRACT

Segmenting an image is the action of assigning pixels of an image into one of the two or more groups. The attributes or characteristics of a pixel are similar to each of the pixels in a group. These pixels are rejoined together to figure out an entire image. It can be performed by different methods which includes clustering techniques, thresholding methods, color based segmentation method, region extraction and edge detection methods, etc.,. Clustering is a procedure of collecting the pixels of an image in which the features like intensity, color, etc., of the pixel are more similar within one cluster than the other cluster. The two major clustering schemes are hard clustering and fuzzy clustering methods. In this paper, Among the fuzzy clustering method, Fuzzy C Means clustering method as well as their extensions and their merits and demerits are also discussed to segment the images from which more details can be retained, when compared with hard clustering methods. The extension of the FCM algorithms improves the robustness to various types of noise as well as performance of the clustering.

Keywords: Image Segmentation, Fuzzy C Means clustering algorithm, Euclidean distance, Mahalanobis distance, FLICM algorithm, Gustafson-Kessel Clustering

Corresponding author

INTRODUCTION

Fuzzy clustering method [8] is defined as each data point should be associated with more than one cluster, where each data point has the partial membership in more than one cluster. With hard clustering, each data point of an image is coupled with only one cluster. So, that the results are usually crisp [7]. The membership value of each data point is associated with one cluster. Hard clustering is also known as a crisp clustering method [8]. Still, hard clustering the effectiveness will be degraded in some situations, because of the issues like poor contrast, intensity overlapping, noise and inhomogeneities of intensity. The partial membership concept is described from the Fuzzy set theory [10]. Fuzzy clustering has been generally applied in image segmentation as well as clustering [18] [19] [20]. Image segmentation plays a vital role in many application fields like Medical imaging, geographical imaging, robot vision, object recognition.

Stelios Krinidis and Vassilios Chatzis [1] suggested the FLICM algorithm an extension of the ordinary FCM algorithm to strengthen the execution of the clustering because FLICM added both local spatial and gray similarity measures, it can have capacity to beat the downsides of conventional FCM. Finally, they proved that the FLICM algorithm provides an efficient and effective clustering of image with robustness to any types of noise and image detail preservation without using any parameter.

Maoguo Gong et al. [2] Proposed a Reformulated FLICM algorithm for grouping the altered and unaltered region from the fusion of difference image. Difference image is generated based on the wavelet image fusion technique by utilizing the relevant data from both the corresponding set of images. Background information was established by log ratio image and changed regions information acquired by the mean ratio image. The DWT fusion method operates at pixel level of the source image. RFLICM includes the both similarity information entirely, but a novel fuzzy way of spatial related information is included to diminishing the issues of speckle noise.

Hire Gayatri Ashok et al. [3] Performed a survey on change detection in SAR images. They discussed some pre-processing methods to generate the difference image, then FLICM and improved FLICM algorithms to detect the changed regions.

Niladri Shekhar Mishra et al. [4] Discussed about FCM and Gustafson Kessel clustering and two validity measures are used to evaluate the execution of both clustering results. They considered a new strategy for adding local information. Each pixel in the image is highly overlapped with neighbors. So, it is hard to isolate the altered and unaltered areas. The proposed clustering algorithm separates the pixel of the difference image into altered and unaltered regions. The objective functions of the FCM and GKC clustering algorithms can be minimized by the use of optimization techniques such as genetic algorithm and simulated annealing. Finally, GKC method separate clusters with different shapes and improves the segmentation quality than FCM which separates the cluster only with a spherical shape.

Sandeep et al. [5] suggested a fuzzy local c means algorithm as a variation of FCM to achieve the robustness to various types of noise and protecting details of the image and also enhance the performance of the clustering.

Weiling Cai , Songcan Chen et al. [6] Suggested a FGFCM method which mitigates the disadvantages of existing algorithms. FGFCM adds a fuzzy local measures which aim to ensure the toughness to noise and thus maintains image details.

Xuemei Zhao et al. [11] discussed a Mahalanobis distance norm based FCM clustering algorithm for segmenting an image. Euclidean norm is used to measure the dissimilarity between the data points and the prototypes of cluster center, because it only distinguish the average details of a cluster. So, FCM is extremely reactive towards noise as well as does not find cluster covariance. Here, they proposed an Mahalanobis based FCM to measure the dissimilarity. A newfangled regularization term is incorporated into MFCM's objective functions, which reflects the cluster's covariance. The efficacy of the MFCM method is verified experimentally. Mahalanobis distance utilizes the mean and covariance of the cluster through a covariance matrix. They experimentally showed the enhanced segmentation result of MFCM which performs better than traditional FCM.

Mohamed N. Ahmed et al. [12] suggested a Fuzzy C-Means with Spatial constraints algorithm (FCM_S) for MRI and SAR images like data to remunerate the assessment of intensity inhomogeneities. In the FCM_S distance between a pixel point to a center of the cluster and its neighbor pixel point to the cluster center are calculated through the use of Euclidean norm. The weighted mean value of those distances measures the dissimilarity between the cluster centers and pixel points. The neighbor pixel progress just a regularizer and bias the output in order to achieve piecewise homogeneous labels. Regularization is useful while segmenting the image which is affected by impulse noise. In FCM_S, at each iterative step Euclidean distance from the neighboring pixel to the center of the cluster is computed. Finally, the outcomes show that the proposed bias corrected FCM algorithm outperforms the other standard FCM algorithm. Even though, neighborhood details are incorporated into their objective functions, it is a time consuming process and also inadequate to outliers.

Synthetic aperture radar images are corrupted by the occurrence of speckle noise becomes hard to segment due to its multiplicative nature. However, some existing algorithms such as FCM and their extensions can produce satisfactory segmentation results as well as sturdiness to Gaussian and impulse noise, but, these methods are not flexible to SAR images with speckle noise. Kernel Fuzzy C Means (ILKFCM) was presented by Deliang et al. [14] to conquer the problem of existing FCM related algorithms on segmenting the SAR images. Kernel Fuzzy clustering algorithm objective function adds a fuzzy weighted factor in which location and intensity details of all neighbor pixels are taken into the account simultaneously for segmentation of SAR images. In this algorithm, the original image is used for clustering and preprocessing steps would be ignored. Original synthetic aperture images are decomposed using wavelet decomposition method. From the wavelet decomposition method, images are divided into varying frequency channels. Its energy features denotes the details about the spatial frequency of the original image which is extracted for clustering. Feature similarities are determined using kernel distance which is more strong to outliers. Weight factors provide sturdiness to speckle noise. ILKFCM uses few parameters. The results demonstrated that the proposed ILKFCM is efficient for segmentation of SAR images.

Maoguo Gong et al. [13] described a new approach for detecting changes in SAR images by the FCM method with the energy function of Markov random field. The experimental analysis shows that the FCM with energy function will reduce the result of speckle noise.

The segmentation result of FCM_S and its variants are enhanced by utilizing the kernel distance based methods instead of using the Euclidean distance measure was suggested by Songcan Chen et al [15]. The objective to acquire the kernel function is to induce the new distance measure rather than the Euclidean distance measure for segmentation as well as to keep the computational simplicity.

RFLICM algorithm is enhanced by including the kernel metrics and weight fuzzy factor measures both are parameter free which is termed as KWFLICM. RFLICM still lacks to eliminate the impact of spatial constraints on the relation among the central pixel and neighborhood pixels. The objective function adds a fuzzy weighted factor in which location and intensity details of all neighborhood pixels are taken into the account simultaneously for segmentation of images. The fuzzy weighted factor mainly used for adaptively adjusting the spatial neighborhood relationship. The result are experimentally proved by Maoguo Gong et al. [16] that the clustering based on the kernel method improves the clustering performance and thus provides sturdiness to noise.

Fuzzy C Means with spatial constraints as well as its variants using the kernel method gives more sturdiness to outliers and noise when compared among the FCM_S along with its variants clustering algorithms without using kernel methods [17].

ALGORITHMS

This section deals with different fuzzy clustering methods and their merits and demerits.

FCM

The most conventional fuzzy clustering method used in image segmentation is a **F**uzzy **C** **M**eans algorithm [1] [4] [8] [11]. FCM operates effectively for noiseless images. It is an iterative algorithm which gives an optimal number of cluster groups by minimizing the sum of squared error which is then weighted by the membership of y_i in the cluster j, the objective function Q is defined as,

$$Q = \sum_{i=1}^{M}\sum_{j=1}^{N} P_{ji}{}^{s} \left\| y_i - c_j \right\|^2 \qquad (1)$$

Where y_i represent the data points Y= {y_1, $y_{2 \ldots}$ y_M}, M specifies the amount of data points, N represents the size of clusters required in the range of $2 \le N < M$, P_{ji} symbolize membership of y_i in the j^{th} cluster. $\|y_i - c_j\|$ in equation (2) measures closeness between the data point y_i, and the j^{th} cluster center. s specifies the amount of fuzziness.

$$d_{ji} = \| y_i - c_j \| = \sqrt{(y_i - c_j)^2} \text{ is the Euclidean distance norm.} \qquad (2)$$

The objective function (1) can be acquired from the following iterative steps:

1. Initially fix the size of cluster N, fuzzifier coefficient s, and termination condition **€**.
2. Fuzzy partition matrix P_{ji} will be initialized first
3. Loop counter is set to zero (i.e., k=0)
4. Determine the cluster Centers c_j

$$c_j = \frac{\sum_{i=1}^{M} P_{ji}{}^{s} * y_i}{\sum_{i=1}^{M} P_{ji}{}^{s}} \qquad (3)$$

5. Evaluate the membership partition matrix

$$P_{ji} = \frac{1}{\sum_{k=1}^{N}\left(\frac{\|y_i - c_j\|}{\|y_i - c_k\|}\right)^{\frac{2}{s-1}}} \qquad (4)$$

6. If max {$P_{ji}{}^{k} - P_{ji}{}^{k+1}$} < **€** then stop otherwise repeat from step 4.

The Advantage of the FCM is it can able to preserve more information, its simplicity and less computational time. The effectiveness of FCM is accomplished from the value of fuzzifier coefficient. It uses Euclidean distance that gives clusters with only circular shape. To remunerate this issue of FCM algorithm, smoothing filter methods have been suggested in [12], but the image details may be subjected to lost. The shortcoming of FCM is extremely reactive to noise and not considered any spatial context related data. So as to enhance the performance of the quality of segmentation, several researchers include spatial context related information into the standard FCM algorithm.

FCM_S

The modified objective task of the conventional FCM algorithm is given by presenting a term which permits the pixels that are labelled have an impact on labelling of its immediate neighborhood pixels of the image [1] [6].

FCM_S [12] objective function is defined as:

$$Q = \sum_{i=1}^{M}\sum_{j=1}^{N} P_{ji}{}^{s} \|y_i - c_j\|^2 + \frac{\alpha}{M_R}\sum_{i=1}^{M}\sum_{j=1}^{N} P_{ji}{}^{s} \sum_{r \in M_i} \|y_r - c_j\|^2 \qquad (5)$$

Here, gray value of the pixel i denoted by y_i, M represents the amount of pixel count, the centroid of the respective cluster is represented by c_j, P_{ji} denotes membership value of the pixel i in cluster j, N specifies the size of clusters, M_R as its cardinality, y_r represents neighbor pixel of y_i, M_i is a set of neighbors around pixel y_i. α specifies a parameter to regulate the neighbors term. It is necessary to satisfy that the membership value of a single data point should be unity. $\sum_{i=1}^{N} P_{ji} = 1$

The membership matrix (P_{ji}) will be calculated by

$$P_{ji} = \frac{(\|y_i - c_j\|^2 + \frac{\alpha}{M_R} \sum_{r \epsilon M_i} \|y_r - c_j\|^2)^{1/s-1}}{\sum_{k=1}^{N}(\|y_i - c_k\|^2 + \frac{\alpha}{M_R} \sum_{r \epsilon M_i} \|y_r - c_k\|^2)^{1/s-1}} \qquad (6)$$

Update the cluster centre (c_j) by using,

$$c_j = \frac{\sum_{i=1}^{M} P_{ji}^{s} \left(y_i + \frac{\alpha}{M_R} \sum_{r \epsilon M_i} y_r\right)}{(1+\alpha) \sum_{i=1}^{M} P_{ji}^{s}} \qquad (7)$$

One shortcoming of FCM_S is that at each iterative step labelling of the neighborhood is computed. So it is a time consuming process. The execution time of FCM_S was reduced by the introduction of its variants such as FCM_S1, FCM_S2 algorithms.

FCM_S1

FCM_S1 uses extra mean filtered image instead of neighbor terms in FCM_S [1] [6]. Their objective function Q is expressed as follows

$$Q = \sum_{i=1}^{M} \sum_{j=1}^{N} P_{ji}^{s} \|y_i - c_j\|^2 + \alpha \sum_{j=1}^{N} \sum_{r \epsilon M_i} P_{ji}^{s} \|\bar{y}_r - c_j\|^2 \qquad (8)$$

Where $\overline{y_r}$ denotes the neighboring pixels average around y_r. Computational time is reduced by replacing $\frac{1}{M_R} \sum_{r \epsilon M_i} \|y_r - c_j\|^2$ with $\|\bar{y}_r - c_j\|^2$.

The membership matrix (P_{ij}) will be calculated by,

$$P_{ji} = \frac{(\|y_i - c_j\|^2 + \alpha \|\overline{y_i} - c_j\|^2)^{1/s-1}}{\sum_{k=1}^{N}(\|y_i - c_k\|^2 + \alpha \|\overline{y_i} - c_k\|^2)^{1/s-1}} \qquad (9)$$

Update the cluster centre (c_j) by using,

$$c_j = \frac{\sum_{i=1}^{M} P_{ji}^{s} (y_i + \alpha \overline{y_i})}{(1+\alpha) \sum_{i=1}^{M} P_{ji}^{s}} \qquad (10)$$

The goal of FCM_S1 is to get both the source and its equivalent neighbors average filtered images should have the same result. Its execution time was significantly minimized and also enhances the sturdiness towards Gaussian noise. Although, the FCM_S1 be incompatible for image which is attacked by salt and pepper noise (impulse noise).

FCM_S2

FCM_S2 overcomes the disadvantage of FCM_S1 by using median a nonlinear process of the filtered image rather than average filtered image during clustering to enhance the sturdiness to impulse noise. It provides robustness to impulse noise and also reduces the execution time [1] [6]. In both FCM_S1 and FCM_S2, essential parameter α was used which balance the exchange among the source with its equivalent filtering images. The task of selecting the value for a parameter becomes difficult.

ENFCM

In this **En**hanced **F**uzzy **C M**eans, clustering done on the gray histogram rather than using pixels of the newly produced linearly weighted sum image [1] [6]. Linearly weighted sum image (ω) can be obtained from together source image along with the average filtered image. The amount of pixels in an image is greater than G.

$$\omega_i = \frac{1}{1+\alpha}\left(y_i + \frac{\alpha}{M_R}\sum_{j\in M_i} y_j\right) \qquad (11)$$

Where ω_i is the i[th] pixel's gray value of the image ω, M_i indicates the group of neighbors (y_j) around y_i.

Their objective function is written as

$$Q = \sum_{i=1}^{G}\sum_{j=1}^{N} \gamma_i P_{ji}^{\,s} \parallel \omega_i - c_j \parallel^2 \qquad (12)$$

G be the amount of gray value of image ω which is lesser than the amount of its pixels M. i.e M > G, γ_i denotes no.of. pixels with the gray value equivalent to i, here i = 1,..,G. Therefore, $\sum_{i=1}^{G}\gamma_i$ =M along with the condition $\sum_{j=1}^{N} P_{ji}^{\,s} = 1$. Q is minimized by satisfying the following equations for computation of the cluster centers C_j and a membership matrix P_{ji} :

$$P_{ji} = \frac{(\omega_i - c_j)^{2/s-1}}{\sum_{k=1}^{N}(\omega_i - c_k)^{2/s-1}} \quad (13) \qquad C_j = \frac{\sum_{i=1}^{G}\gamma_i P_{ji}^{\,s}\omega_i}{\sum_{i=1}^{G}\gamma_i P_{ji}^{\,s}} \quad (14)$$

Their Computational time is crucially reduced and also provides better segmentation result when compared to FCM_S. The quality of the segmented result may vary depends on the parameter (α) value and filtering methods used.

It lacks the robustness to mixed noise and also difficult to select parameter values with no prior knowledge. The parameter actually balances the noise and image details. If the value of the parameter chosen to be small, then it protects the sharpness and details of an image. When the value of the α is larger then the method can able to eliminate the noise.

FGFCM (**F**ast **G**eneralized **F**uzzy **C M**ean)

In this algorithm clustering was done on the gray histogram of the non-linearly weighted sum image (ω) which can be produced from the source image along with its local information neighborhood [1] [6]. This algorithm adds the similarity measures S_{ij} which contains the local spatial ($S^s{}_{ij}$) and gray level ($S^g{}_{ij}$) image information to enhance the clustering result. S_{ij} parameter is to replace the defects of using common parameter α in FCM_S, its variants and EnFCM. Its computational time is very small and effectively enhances the quality of segmented image.

$$S_{ij} = \begin{cases} S^s{}_{ij} \times S^g{}_{ij} & i \neq j \\ 0 & i = j \end{cases} \qquad (15)$$

$S^s{}_{ij}$ and $S^g{}_{ij}$ denotes the spatial and gray level relationship respectively.

$$S^s{}_{ij} = exp\left(\frac{-max(|R_i - R_j|,|T_i - T_j|)}{\lambda_s}\right) \quad (16) \quad \& \quad S^g{}_{ij} = exp\left(\frac{-\|y_i - y_j\|^2}{\lambda_g \times \sigma_{g\_i}{}^2}\right) \quad (17)$$

Where y_i represent the gray value of image pixel, pixel i denote the central pixel of the picture matrix and pixel j denotes the group of neighbors of image matrix around i[th] pixel. (R_i,T_i) are the spatial coordinates of pixel i. ξ_i denotes i[th] pixel gray level value of the image ω , N_i represents group of neighbors, S_{ij} measures the similarity between i[th] and j[th] pixel. y_j represents gray-values of the neighbors around the pixel y_i.

$$\sigma_{g\_i} = \sqrt{\frac{\sum_{j\epsilon N_i}\|y_i - y_j\|^2}{N_R}} \quad (18) \qquad \omega_i = \frac{\sum_{j\epsilon N_i} s_{ij} \times y_j}{\sum_{j\epsilon N_i} s_{ij}} \quad (19)$$

In FGFCM algorithm λ_s *is fixed and* λ_g controls the algorithm. λ_s is fixed earlier, when the image matrix window size is determined and changes the pixel value in accordance with spatial distance from the central pixel. The role of Sij in FGFCM ensures the noise immunity and keeps more details about the image than ENFCM because the Sij value changes from every pixel.

All the above discussed algorithms use a common parameter (α in FCM with spatial constraints and its variants, Enhanced Fuzzy C Means or λ_s,λ_g in FGFCM) to control trade off among the source image along with its equivalent filtered image. It is not an easy task to choose the parameter value because the parameter ought to balance the insensitive to noise as well as maintains the image details. If (α or λ_g) value has to be chosen small, then it preserves image details and if it is larger, then it can eliminate the noise. All the above algorithms may be subject to lost details of the original image based on the methods used to produce new images and however, they are insensitive to noise towards some degree, still need sufficient amount of hardness towards noise and outlier without prior knowledge of noise.

All the above algorithms follow these steps to minimize the objective function:

1. Fix the no. of. cluster N, stopping criteria € and fuzzifier value s
2. In FCM_S1 algorithm calculate mean filtered image, for FCM_S2 find median filtered, for ENFCM and FGFCM determine linearly weighted sum image (ω) by equation(11) and non – linearly weighted sum image (ω) using equation (19) respectively.
3. Fuzzy partition matrix P_{ji} will be initialized randomly and loop counter is set to k=0.
4. Update the prototype of FCM_S by the equation (7), FCM_S1 &FCM_S2 using equation (10) and equation (14) for ENFCM &FGFCM
5. Compute the partition matrix by the equation (6) for FCM with spatial constraints, using the equation (9) for FCM_S1 and FCM_S2 or equation (13) (ENFCM &FGFCM)
6. If max $\{P^{k} - P^{k+1}\}$ < € then terminate. Otherwise, reiterate steps 3 – 4 until the stopping criteria is satisfied.

FLICM

Fuzzy **L**ocal **I**nformation **C M**eans clustering algorithm [1] is a completely free of using any parameter because of the introduction of novel fuzzy factor [2] [3] which handles the problem of parameter selection in the above discussed algorithms. A novel fuzzy factor contains local information similarity measures which provide insensitive to noise and details of an image preservation. The fuzzy factor should have some features

- To be independent to different types of noise,
- There is no need of any parameter selection,
- Introduction of fuzzy factor balances the robustness to noise and preserves image details
- Uses original image to avoid pre-processing step that could lost image details
- To control the impact of neighbor pixels depends on the spatial distance from the central pixel.

These characteristics make FLICM [5] enhances the clustering performance.

Fuzzy factor F_{ki} defined as follows,

$$F_{ki} = \sum_{\substack{j \in M_i \\ i \neq j}} \frac{1}{d_{ij} + 1} \left(1 - P_{kj}\right)^s \left\|y_{j} - c_k\right\|^2 \qquad (20)$$

Here, i denotes a central pixel value of the image matrix window (3X3), k denotes the reference cluster and M_i is a group of neighbors falling into the image matrix window in the region of the pixel i. d_{ij} tells about the Euclidean norm between pixels. P_{kj} in equation (22) represents the membership value of pixel j to the k[th] cluster center. c_k in equation (23) are the prototype of the cluster centers k. S specifies weighting constraint on each value of membership and thus decides the quantity of fuzziness.

Their objective function will be defined in terms of the equation (21)

$$Q = \sum_{i=1}^{M} \sum_{k=1}^{N} [P_{ki}{}^{s} \|y_i - c_k\|^2 + F_{ki}] \qquad (21)$$

$$P_{ki} = \frac{1}{\sum_{j=1}^{N} \left(\frac{\|y_i - c_k\|^2 + F_{ki}}{\|y_i - c_j\|^2 + F_{ji}} \right)^{1/_{s-1}}} \quad (22) \qquad \& \qquad c_k = \frac{\sum_{i=1}^{M} P_{ki}{}^{s} \times y_i}{\sum_{i=1}^{M} P_{ki}{}^{s}} \quad (23)$$

The steps to perform the FLICM algorithm:

1. Number of clusters are fixed (N), fuzzifier parameter (s), termination condition (€)
2. Fuzzy Partition matrix P_{ij} will be initialized randomly
3. Initializing the loop counter (k = 0)
4. Computing the cluster prototype in equation (23)
5. Determine the fuzzy partition matrix using equation (22)
6. If max { $P^{k} - P^{k+1}$ } < € then terminate. Otherwise, set k=k+1 and repeat from the 4[th] step.

Since, FLICM uses Euclidean distance measures as in FCM but, FLICM are more robust because of introduction of fuzzy factor. The introduced fuzzy factor controls the impacts of noisy pixels and thus becomes further strong to outliers. It also converges the membership value of noisy pixel and no-noisy pixels with a comparable value, so it disregards the influence of the noise. By each iteration and each pixel the value of F_{ki} will be changing rather than using same value thus preserves more image information. FLICM preserves more image details; robustness to noise and outliers, without using the parameter clustering is being performed directly on the source image.

RFLICM

The fuzzy factor F_{ki} in FLICM has local spatial as well as local gray information as a similarity measure. The spatial information is represented by spatial distance, which changes consequently with spatial distance from the central pixel. F_{ki} could not able to effectively restrain the influence of noisy pixels. So as to compensate this, **R**eformulated **F**uzzy **L**ocal **I**nformation **C M**eans algorithm [2] is presented, the local variation coefficient is embraced to change the local spatial information like similarity measure in FLICM [3]. The local variation coefficient C_u is written as follows,

$$C_u = \frac{\text{var}(y)}{(\bar{y})^2} \qquad (24)$$

Where var(y) represents the intensity variance, $(\bar{y})$ represents mean value of the image. By using the local coefficient of variation, fuzzy factor balances degree of membership value among central pixel and its neighbor. In the event that there is a dissimilarity among the results of local variation coefficient that are produced from the central pixel and its neighbor pixel, then weight added of the neighbor pixels in $\tilde{F}_{ki}$ thing will be improved to suppress the influence of noisy pixels effectively and be more robust to outliers.

Fuzzy factor $\tilde{F}_{ki}$ defines as follows,

$$\acute{F_{ki}} = \begin{cases} \sum_{j \in M_i} \frac{1}{2 + min\left(\left({C_u}^j/_{C_u}\right)^2, \left({C_u}/_{{C_u}^j}\right)^2\right) \times (1 - P_{kj})^s \|y_j - c_k\|^2}, & if\ {C_u}^j \geq \overline{C_u} \\ \sum_{j \in M_i} \frac{1}{2 - min\left(\left({C_u}^j/_{C_u}\right)^2, \left({C_u}/_{{C_u}^j}\right)^2\right) \times (1 - P_{kj})^s \|y_j - c_k\|^2}, & if\ {C_u}^j < \overline{C_u} \end{cases} \quad (25)$$

Where C_u represents central pixels local coefficient of variation, ${C_u}^j$ denotes the neighboring pixels local variation coefficient, $\overline{C_u}$ denotes the average of ${C_u}^j$. RFLICM follows the same step performed by the FLICM algorithm to minimize the objective function, but the fuzzy factor only varies from spatial relationship to the local coefficient of variation.

GUSTAFSON-KESSEL CLUSTERING

This method utilizes fuzzy covariance matrix (Z_i) [7] of the cluster center to measure the distance between the clusters. GKC [9] is used to detect clusters of different shapes in one dataset. It uses Mahalanobis distance D_{ikF_i} instead of using Euclidean distance d_{ij} for clustering purpose. FCM employs Euclidean distance so detect only spherical shape, but it cannot identify clusters with non spherical shapes like ellipsoidal, etc.,. By using GKC we can get an ellipsoidal shape cluster. Each cluster could be automatically adopted its shape through the use of its own norm-inducing matrix F_i.

The fuzzy covariance matrix [4] of the cluster i is defined by equation (26),

$$Z_i = \frac{\sum_{k=1}^{M} (P_{ik})^s \times (y_k - c_i) \times (y_k - c_i)^T}{\sum_{k=1}^{M} (P_{ik})^s} \quad (26)$$

Norm inducing matrix is written by equation (27),

$$F_i = [\rho_i \times determinant(Z_i)]^{1/\eta} \times Z_i^{-1} \quad (27)$$

Where η is the dimension of inputs. ρ_i specifies a constant which catch up the proper shape of the related clusters. The Mahalanobis distance between the clusters is given by equation (28) and membership partition matrix are computed by using equation (29),

$$d_{ikF_i} = \sqrt{(y_k - c_i) F_i (y_k - c_i)^T} \quad (28) \quad \& \quad P_{ik} = \frac{1}{\sum_{j=1}^{N} \left(\frac{d_{ikF_i}}{d_{jkF_i}}\right)^{2/s-1}} \quad (29)$$

The objective function will be explained interms of equation (30),

$$Q = \sum_{i=1}^{N} \sum_{k=1}^{M} {P_{ik}}^s \times D_{ikF_i} \quad (30)$$

Here, $D_{ikF_i} = {d^2}_{ikF_i}$, GKC algorithm provides better clustering performance when compared to FCM.GKC extracts clusters with different shapes and requires only less computational time. The value of ρ_i affects the result of Gustafson Kessel clustering (GKC) algorithm.

MFCM (Mahalanobis distance based **FCM)**

The conventional FCM uses Euclidean based norm to measure the distance between the pixels. But, the objective function of the MFCM [11] based on a Mahalanobis norm to determine the dissimilarity between the mean of the data points and covariance of the cluster points and also with the newly added regularization term connected to cluster covariance which aims to decide the covariance matrix. Euclidean distance based FCM is highly reactive to noise. To conquer this Mahalanobis norms are used.

Mahalanobis distance is calculated by equation (31),

$$d_{M\_ik} = (y_k - c_i) \times (y_k - c_i)^T \times Z_i^{-1} \quad (31)$$

Here, Z_i denotes the covariance matrix of the respective ith cluster. while Z_i is assigned as a unit matrix, then Mahalanobis be reduced near the Euclidean norm. If the Euclidean measure is used for measuring the dissimilarity, then the algorithm produces only spherical shapes. If Mahalanobis is used, then it derives cluster with different shapes. It is obvious, that the computed Euclidean distance is same for each and very cluster, but it should compute the various Mahalanobis distance for each cluster. Because, every one cluster covariance is considered into the account for Mahalanobis. For example, let us consider the three clusters like white, brown and orange color cluster. Euclidean distance computed is same for three clusters, the Mahalanobis norm for white cluster is differed from the brown and orange color cluster. The Mahalanobis norm gives more accuracy than the Euclidean norm to segment the images.

The MFCM objective function is defined in equation (32),

$$Q_{MFCM} = \sum_{i=1}^{M}\sum_{k=1}^{N} P_{ik}\, d_{M_{ik}} + \tau \sum_{i=1}^{M}\sum_{k=1}^{N} P_{ik} \, log|Z_k| + \tau \sum_{i=1}^{M}\sum_{k=1}^{N} P_{ik}\, log\left(\frac{P_{ik}}{\rho_k}\right) \quad (32)$$

In the above equations, second terms, express the regulation factor, $|Z_k|$ denotes the determinant of the covariant matrix. Partial derivation is used to compute the covariance. ρ_k controls the amount of the cluster. Mahalanobis Fuzzy C Means clustering performs better than the other existing methods.

The procedure of the MFCM can be explained as follows:

1. Amount of clusters are fixed (N), fuzzifier parameter (s), termination condition (€)
2. Fuzzy Partition matrix P_{ij} will be initialized randomly
3. Initializing the loop counter (k = 0)
4. Determine the cluster mean c_k using equation (33) and covariance matrix (Z_i) of the cluster by using equation (34)

$$c_k = \frac{\sum_{i=1}^{M} P_{ki}^{s} \times y_i}{\sum_{i=1}^{M} P_{ki}^{s}} \quad (33) \quad \& \quad Z_i = \frac{\sum_{k=1}^{M} (P_{ik})^s \times (y_k - c_i) \times (y_k - c_i)^T}{\tau \times \sum_{k=1}^{M} (P_{ik})^s} \quad (34)$$

5. The fuzzy partition matrix will be computed by the equation (35),

$$P_{ik} = \frac{\rho_k \times exp\left(-\frac{d_{M\_ik} + \tau log|Z_k|}{\tau}\right)}{\sum_{k'=1}^{N} \rho_{k'} \times exp\left(-\frac{d_{M\_ik'} + \tau log|Z_{k'}|}{\tau}\right)} \quad (35)$$

6. If max { $P^{k} - P^{k+1}$ } < € then terminate. Otherwise set k=k+1 and repeat from the 4th step.

CONCLUSION

The robustness and efficient performance of the above discussed algorithms are compared by means of average segmentation accuracy measure. When compared to FCM and various extensions of FCM algorithms, FLICM algorithm improved the performance of the clustering. FCM is sensitive to noise. FCM_S and its variations, ENFCM, FGFCM enhance the performance and provides robustness to noise to a degree, still need enough robustness, a common parameter is utilized to balance the sturdiness to noise and image detail preservation. FLICM works well for all types of noise balance the robustness to noise and extracts more image information without using any parameters. Compared to FLICM, the RFLICM fuzzy factor will suppress the impacts of noisy pixels effectively and thus provides enough insensitiveness to noise and maintains more information about the image. In FCM, selecting the fuzzifier value may lead to affect the clustering results and still uses Euclidean distance norm, it recovers cluster with only spherical shape, but the GKC can able to discover the cluster with different shapes with the help of $\square_{\square}$ value and also yields better clustering result. MFCM would not think the information about the neighbors in the image and thus provides more efficient segmentation results than the entropy based FCM. RFLICM utilizing a Euclidean distance, but an improved version of FLICM such as KWFLICM utilizes the kernel method which is non-Euclidean in structure. KWFLICM seems to be more robust to independent types of noise.

DECLARATION

The authors declared that the manuscript is original and is not published or communicated for publication elsewhere either in part or full.

REFERENCES

[1] Krinidis S, Chatzis V, IEEE Transactions on Image Processing, 2010; 5: 1328-1337.
[2] Gong M, Zhou Z, Ma J, IEEE Transactions on Image Processing, 2012; 4: 2141-2151.
[3] Hire Gayathri Ashok, Patil D.R, International Journal of computer Applications, 2014; 2: 4-7.
[4] Mishra N.S, Ghosh S, Ghosh A, Applied Soft Computing, 2012; 8: 2683-2692.
[5] Sandeep Kumar Mekapothula, Jai Kumar V, International Journal of Scientific and Engineering Research, 2011; 12: 1-7.
[6] Cai W, Chen S, Zhang D, Pattern Recognition, 2007; 3: 825-838.
[7] Ghosh A, Mishra N.S, Ghosh S, Information Sciences, 2011; 4: 699-715.
[8] Timothy J.Ross, Fuzzy Logic With Engineering Applications, University of New Mexico,USA, Second Edition, 2004.
[9] Krishnapuram R, Kim J, IEEE Transactions on Fu,2004zzy Systems, 1999; 4: 453-461.
[10] Lin Zhu, Fu-Lai Chung, Shitong Wang, IEEE Transactions on Systems, Man and Cybernetics, 2009; 3: 578-591.
[11] Zhao X, Li Y, Zhao Q, Digital Signal Processing, 2015; 8-16.
[12] Ahmed M.N, Yamany S.M, Mohamed N, Farag A.A, Moriarty T, IEEE Transactions on Medical Imaging, 2002; 3: 193-199.
[13] Gong M, Su L, Jia M, Chen W, IEEE Transactions on Fuzzy Systems, 2014; 1: 98- 109.
[14] Deliang Xiang, Tao Tang, Canbin Hu, Yu Li, Yi Su, IEEE Geoscience and Remote Sensing Letters, 2014; 7: 1290-1294.
[15] Chen S, Zhang D, IEEE Transactions on Systems, Man, and Cybernetics, 2004; 4: 1907-1916.
[16] Gong M, Liang Y, Shi J, Ma W, Ma J, IEEE Transactions on Image Processing,2013; 2: 573-584.
[17] Kuo-Lung Wu, Miin-Shen Yang, Pattern Recognition, 2002; 10: 2267–2278.
[18] Noordam J, Van den Broek W, Buydens L, Proceedings of International Conference on Pattern Recognition, 2000; 462–465.
[19] Yang M.S, Hu Y.J, Lin K, Lin C.C, Magnetic Resonance Imaging, 2002; 2: 173– 179.
[20] Karmakar G, Dooley L, Pattern Recognition Letters, 2002; 10: 1215–1227.