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Algorithm of Generation of Scale-Free Network at Realization Virus Attacks on Model Chiang Lu.

Elena Sergeevna Sokolova*, Nikolai Ilyich Barannikov, Igor Leonidovich Bataronov, and Vladimir Ivanovich Belonozhkin,

Voronezh State Technical University, Russian Federation, 394026, Voronezh, Moskovsky prospect, 14.

ABSTRACT

In this paper there is a generation model, which satisfied a power law, also the weight of each layer is considered. Physicists investigate the statistical properties of networks, such as laws of the number of nodes distribution links, unlike social scientists. By studying these questions, it was found a large number of amazing and intriguing properties of real networks, which weren't studied by mathematicians and sociologists. These properties have stimulated the development of new theories, models, measurements, revealing new fundamental networking features. Physics journals publish vast majority of the most important works in this field now. The term "complex networks" appeared at the beginning of this century and refers to networks with more complex architecture than the classical random network with a given number of nodes and links, or lattice in crystals. Typically, such networks have a small number of sites with a large number of bonds - hubs, which largely determine the properties of these networks. It turned out that most real networks (biological, technical, social) are complex. Recently, modeling of various epidemics in the network became popular. Mathematical analysis of such epidemics can simulate them in artificial conditions, but also analyze the possible damage and the risk of this epidemic. This subject has been widely used and it is actually at the moment.

Keywords: free – scale network, epidemic, risk, generation, the Lu Chang model, power law distribution.

Corresponding author

INTRODUCTION

Global development of the Internet and social networks has attracted a big attention of scientists: there are many papers about that [1-4, 8, 19, 20] which consider different models of information epidemics development [6, 18]. There are paradigmatic layered models of epidemics in scale-free networks in papers [7, 9-16], however, in test networks is not considered the importance of nodes.

The network paradigm with physical concepts and methods has become a major and very effective tool for the study of real complex systems in recent years. Specialists in graph theory and sociology studied network structure earlier.

At present there are a considerable number of works that examine different models of building relationships in networks. There is a large number of works dealing with various models of information epidemics. There are a small amount of works about modeling epidemics in scaleless networks at present time.

Math equations

Barabási and Albert proposed a simple and elegant model of the origin and evolution of scaleless networks. They showed that two conditions are necessary [1] for the emergence of scaleless networks:

1. Growth. Starting with a small number of nodes m_0 , at each time step a new node is added with $m,\left(m \le m_0\right)$ links that connect the new node with m different existing nodes;
2. Preferential attachment. When nodes to which the new node joins are selected, it is assumed that the probability P, with which the new node will be connected to the existing node *i*, depends on the number of links wich this node is already has with other nodes, so that: $P\left(q_i\right) = q_i \sum_j q_j$.

Scale-free network is the phenomenology manifestation of critical phenomena in complex systems, because their structure obeys a power law [12].

We shall look at the model, where the significance of nodes depends on a value of information, which is generated, processed and transmitted by the node, and a number of node communications. To define the weight for nodes we shall use the Lu Chang model. This model is defined by a set of weights $w = \left(w_{1,..,} w_n\right)$, which displays an expected sequence of powers. The probability of finding an edge between *i* and *j* is $w_i w_j / \sum_k w_k$, where $k = \frac{\beta - 2}{\beta - 1} dn^{1/(\beta-1)}$. They admit loops from *i* to *i*, so the expected degree of a vertex will equal:

$$\sum_J \frac{w_i w_j}{\sum_k w_k} = w_i. \qquad (1)$$

Of course it has a sense if $max_i w_i^2 < \sum_k w_k$.

In the power – law distribution the probability of k - vertex degree is $p_k = k^{-\beta} / \zeta\left(\beta\right)$, where $\zeta\left(\beta\right) = \sum_{k=1}^{\infty} k^{-\beta}$.

The probability of the degree will be $\ge K is \sim B$ and $1/\mathrm{B} = \left(\beta + 1\right)\zeta\left(\beta\right)$. Assuming that weight will decrease, we will receive:

$w_i = K,\ \frac{i}{n} = BK^{-\beta+1}.$

After solution we will get:

$$w_i = \left(i / nB\right)^{-1/(\beta-1)}. \qquad (2)$$

A task is generation of random network graph $G = (V, E)$ with n vertices and degree distribution, which satisfy the power – law distribution with constant $P(d_i = d) = \Theta(d^{-\beta})$, where $\beta > 1, d_i = \deg(v_i), v_i$ – i-vertex of graph G, and with indication of weight for network node by the Lu Chang model.

At the beginning of algorithm there are following parameters:

1. Natural number n is number of tops in graph G;
2. The average degree of vertices $\bar{d}$;
3. The index distribution $\beta > 1$;
4. The natural number $k < n$ is responsible for the lower limit of the distribution;
5. The natural number m is parameter for parallel algorithm work.

At the end there is the graph G with number of tops *n*, which degrees are distributed by the power – law distribution with parameter β .

The generation of graph is necessary to separate into following steps:

1. The generation of random number w_i , which is distributed by the power – law distribution. These numbers are possible to generate as:

1) From the normalization ratio (3) we shall find a constant a (4):

$$\int_k^{n-1} P(w_i = x)\,dx = \int_k^{n-1} ax^{-\beta} dx; \quad (3)\ a = \frac{1-\beta}{(n-1)^{1-\beta} - k^{1-\beta}}. \qquad (4)$$

2) Then the random numbers we shall find with following term:

$$w_i = \left(\left((n-1)^{1-\beta} - k^{1-\beta}\right) * y + k^{1-\beta}\right)^{\frac{1}{1-\beta}}, (5)$$

where y is uniformly distributed random quantity.

2. The definition of numbers $d_i = \mathrm{b} * w_i$.

$$b = \frac{\bar{d}}{\sum_{i=1}^{n} w_i}. \qquad (6)$$

3. The calculation of M :

$$M = \frac{\sum_{i=1}^{n} d_i}{2}. \qquad (7)$$

4. The calculation of the number sequence:

$$D_1 = d_1,\ D_2 = D_1 + d_2,\ D_{i+1} = D_i + d_{i+1}.(8)$$

5. The parallel generation of M edges.

This algorithm of edge generation is presented on the Fig.1. We have a possibility to build a free – scale graph, based on statistic data [5, 9, 17].

A connectivity coefficient of edges β impacts on the free – scale graph building. Free – scale graph with various coefficients of connectivity can obviously be tracked (Fig. 2.).

Consideration of the networks where there is distribution with non-rigid regularity degree of the vertex $P(k)$, different from the lattice constant of networks, is interesting. Moreover, the proportion of vertex k - degree $P(k)$ can be exponential, power, etc.

Such network can be formal to divide into layers by the size K. This approach is quite natural, because the distribution $P(k)$ considers the inherent network nodes. At the same time, network the arch can theoretically connect tops of all levels. The number of layers $\mathrm{M} = \mathrm{k}_{\max} - \mathrm{k}_{\min}$ is determined by the total number of vertices of the network N and $P(k)$ - a power law distribution of degree of its vertices. Thus, k_{min} can reach 1, it is possible to find k_{max} :

$$\frac{1}{\mathrm{N}} = \mathrm{P}(\mathrm{k}_{\max}).$$

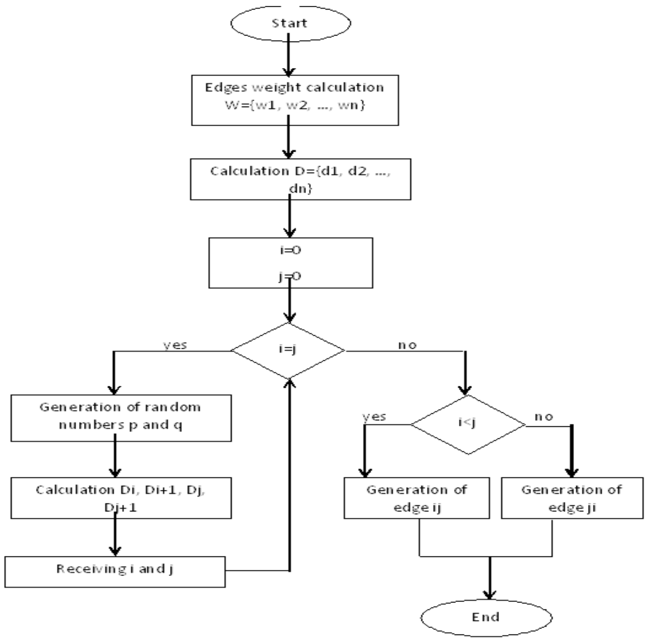
From this we have for the power distribution:

$$\mathrm{k}_{\max} = \sqrt[\gamma]{\mathrm{N}}, \quad \gamma = 2 \div 3.$$

The higher the degree is, the proportion of such peaks in N is smaller.

At the same time, the network should take into account the practical limitations on the dialogue of the various layers representatives. For example, k -layer nodes have the opportunity to interact with tops of specified layers, for example with a plurality $(k \pm r)$ –layers. All this should be done for the most complete variety of epidemiological processes.

Fig. 1. A scheme of edge generation for free – scale graph



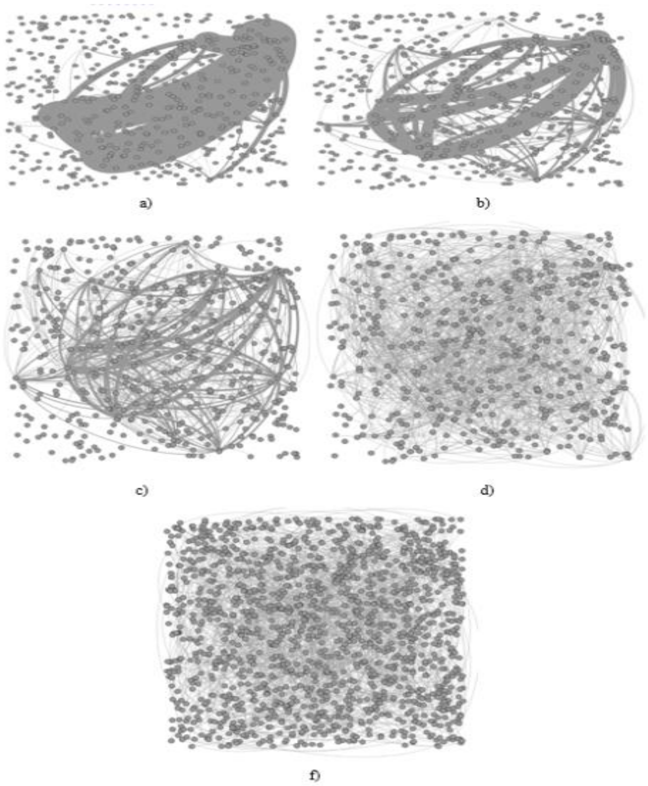


Fig. 2. Visualizing of free – scale graph with parameter a) $\beta = 0.5$ **, b)** $\beta = 0.25$ **, c)** $\beta = 0.1$**, d)** $\beta = 0.01$,
e) $\beta = 0.001$

Splitting scale-free network in layers on top of degree is possible only in the case when nodes are separated on the condition:

$$k_i = \left\{ g_j \in G : \sum_{k=i}^{M} \beta_{ij} = i \right\}$$

where

k_i is layer of the i tops degree;

g_j is top of scale-free graph G;

G is scale-free graph;

β_{ij} is edge between nodes i and j.

Thus, it can be formulated as follows:

For each k_i layer without a large-scale network is true that k_i - is the layer where each vertex g_j with a degree i, belonging to the network of the G, such that the sum of all its links β_{js} for each vertex g_j with other vertices $g_s = \overline{(1, N)}$, equal to i.

Build the adjacency matrix, defines network connection of layers.

$$\begin{pmatrix} \mathrm{k}_{(i_{\min}|j_{min})} & \cdots & \mathrm{k}_{(i_{\min}|j)} & \cdots & \mathrm{k}_{(i_{min},|j_{max})} \\ \vdots & & \vdots & & \vdots \\ \mathrm{k}_{(i|,j_{min})} & \cdots & \mathrm{k}_{i,j} & \cdots & \mathrm{k}_{i,j_{max}} \\ \vdots & & \vdots & & \vdots \\ \mathrm{k}_{(i_{max}|j_{\min})} & \cdots & \mathrm{k}_{i,j} & \cdots & \mathrm{k}_{i_{max},j_{max}} \end{pmatrix}$$

$\mathrm{k}_{i,j}$ is linkage between vertex i and j. If there is a connection, this element takes the value "1", otherwise "0". This matrix has a number of properties. In particular, the sum of the elements in each row is equal to its number of rows, ie .:

$$\sum_{j=i_{min}}^{i_{max}} K(i|j) = i.$$

In addition, the matrix $\|K\|$ is symmetric about the diagonals of [20]:

$$K(i|j) = K(j|i).$$

$$\text{when } i = \left(\overline{min, max}\right),\ j = \left(\overline{min, max}\right).$$

A problem of in layers epidemic development is very interesting. As epidemic can be transferred just between related vertices, the larger links from infected vertex and the higher the degree is, than faster this vertex will spread the infection.

If i is some vertex from n vertices of some layer, then place $i_0 = 1$ and get a formula of vertex degree:

$$w_{ik} = li^{-1/(\beta-1)}, \qquad (9)$$

where $l = \frac{\beta - 2}{\beta - 1} dn^{1/(\beta-1)}$ at $1 \le i \le n$, where the parameter $d = (1/n)\sum_i w_i$ is showing the average value of degree.

Then a total weight of k -layer represents a sum of all vertices weight:

$$w_k = \sum_{i=1}^{n} w_{ik}, \qquad (10)$$

where w_{ik} is weight *i*-node k -layer of free – scale network.

To define damage because of vertex infection, use the formula:

$$U_{i\,t}^{k} = \frac{kw_k}{S_t}, \qquad (11)$$

where w_k is node's weight k - layer of free – scale network;

S_t is the number of uninfected nodes in Poisson's network.

We substitute the formula (10) into (11) and define the damage for k -vertex:

$$U_i^k = \frac{kw_k}{S_t} = \frac{k\sum_{i=1}^{n} li^{-1/(\beta-1)}}{S_t}. \qquad (12)$$

Then an infection of k -layer of free – scale network is:

$$U_{i\,t}^{k} = \frac{k\sum_{i=1}^{n} \frac{\beta_i}{li^{-1/(\beta-1)}}}{S_t}, \qquad (13)$$

where β_i is uniqueness coefficient of i-node.

Accordingly the damage of all network we can calculate by formula:

$$U = \sum_{i=1}^{N} U_{i\,t}^{k} = \sum_{i=1}^{N} \frac{k\sum_{i=1}^{n} \frac{\beta_i}{li^{-1/(\beta-1)}}}{S_t}, \quad (14)$$

The possibility of infection as a link between healthy and infected vertices is calculated by formula:

$$P_i = \alpha_k P(k) = \frac{\alpha_k k x^k{}_m}{x^{k+1}}, \qquad (15)$$

where $P(k)$ is the probability density of power – law distribution.

if to consider clustering degree we will get:

$$P_i = \frac{\alpha_k KP(k)}{\beta_k} = \frac{K\alpha_k kx^k{}_m}{\beta_k x^{k+1}}, \quad (16)$$

where k is the degree of clustering, β_k is correction factor of the degree of clustering. Considering the fact of the dependence of layer danger degree from community quantity, so two factors can be changed

$$P_i = \gamma_k KP(k) = \frac{\gamma_k Kkx^k{}_m}{x^{k+1}}, \qquad (17)$$

where γ_k is common coefficient.

In free-scale networks the possibility of appearing the edge pj of *i*- node with *j*-node for all vertices is the same:

$$P^k_{i\ t}(U) = \sum_{n=1}^{i}\prod_{j=1}^{n} P_i. \qquad (18)$$

The risk is possibility of damage appearing. In the context of epidemiological modeling the risk is a possibility of a single network nodes, a network layer or the all network will be damaged. Thus, you have to calculate damage and risk on the basis of the chosen scale: the network node, the network layer or the all network

We calculate the risk for network node as possibility of damage appearing:

$$Risk^k_{i\ t} = P^k_{i\ t}(U)\cdot U^k_{i\ t}. \qquad (19)$$

We get the formula of infection node risk in free-scale heterogeneous network [10]:

$$Risk^k_{i\ t} = \left(\sum_{n=1}^{i}\prod_{j=1}^{n} p_i\right)\cdot\frac{kw_k}{S_t}. \qquad (20)$$

The risk formula for *k*-layer:

$$Risk^k = \left(\sum_{n=1}^{i}\prod_{j=1}^{n} p_j\right)\cdot U^k_{i\ t} =$$

$$= \frac{\gamma_k Kkx^k{}_m}{x^{k+1}}\cdot\frac{k\sum_{i=1}^{n}\frac{\beta_i}{li^{-1/(\beta-1)}}}{S_t}. \quad (21)$$

And finally the risk formula for all free-scale network:

$$Risk = \left(\sum_{n=1}^{i}\prod_{j=1}^{n} p_j\right)\cdot U + \frac{\gamma_k Kkx^k{}_m}{x^{k+1}} =$$

$$= \sum_{i=1}^{N}\frac{k\sum_{i=1}^{n}\frac{\beta_i}{li^{-1/(\beta-1)}}}{S_t}. \qquad (22)$$

CONCLUSION

We have received analytical expressions for calculation risk of scale-free models such as Chung-Lu, being composed of a large number of nodes, and take effective measures to risk management.

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