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A Variant of Calculation of Harmonic Amplitude in Analyzed Signals.

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ABSTRACT

This article is aimed at revealing of interrelation of current amplitude–frequency spectrum and amplitudes of harmonic components in analyzed signals with subsequent derivation of working equation for detection of amplitudes. With this purpose the article discusses generation of amplitude–frequency spectrum in time, interrelation between harmonic amplitudes and generated spectrum, a variant of working equation for amplitude calculation is proposed with estimation of its error. The interrelation between amplitudes of harmonic components and amplitude–frequency spectrum is obtained provided that the determining contribution to generation of spectrum at preset frequency is made by harmonic components with this frequency.

Keywords: analysis of signals, current amplitude–frequency spectrum, harmonics, harmonic amplitude, interrelation, equation for calculation of harmonic amplitude, equation error.

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INTRODUCTION

Due to development of computing aids the spectral analysis can be widely applied in various fields of human activities. Thus, the spectral analysis is applied for speech recognition, for underwater and surface navigations, object identification, analysis of current state of engineering and biological systems [Murav'ev, 2009; Rybochkin and Yakovlev, 2011; Ivoilova, 2012; Rastegaev et al., 2012; Sorokin et al., 2012; D'yakonitsa et al., 2013; Britenkov and Stepanov, 2014]. Considering for their high importance, significant attention is paid to the researches of properties of spectral analysis [Filatov and Filatov, 2010; Lyubomudrov, 2011; Lyubomudrov and Bashkov, 2011; Antipov et al., 2014; Dzhurovich et al., 2014; Ponomarev et al., 2014].

One of urgent problems in the field of spectral analysis is that of detection of harmonic amplitudes in analyzed signals. This problem should be solved, for instance, upon analysis of current state of engineering items in real time.

This article is aimed at obtaining of interrelation between current amplitude–frequency spectrum and amplitudes of harmonic components in analyzed signals with subsequent deriving of working equation for determination of harmonic amplitudes.

This aim is achieved by means of successive consideration of generation of amplitude–frequency spectrum for harmonics with an arbitrary frequency ω, then the interrelation between the generated spectrum and harmonic amplitudes is established, and, finally, the equation for calculation of harmonic amplitudes using the spectrum is proposed with indication of calculation error.

The researches were performed under the following limitations. It was assumed that the major contribution into generation of spectrum at frequency ω is made by the harmonics with frequency ω. Accordingly, the influence of harmonics with frequencies other than ω on generation of spectrum at frequency ω was not taken into account. In addition, the influence of error of numerical calculations, which are used upon spectrum generation aided by computers on the spectrum generation, was not taken into account.

SOLUTION OF THE PROBLEM AND ITS SUBSTANTIATION

The current amplitude–frequency spectrum for analyzed signal *f = f(t)* in the time interval [0, t] is generated in accordance with the following equation:

$$\Phi(t,\omega) = \sqrt{[\int_0^t f(\tau)\, sin(\omega\tau) d\tau]^2 + [\int_0^t f(\tau)\ \cos(\omega\tau)\, d\tau]^2}\,, (1)$$

where $\Phi(t,\omega)$ is the generated amplitude–frequency spectrum, *t* is the current time, ω is the cyclic frequency, τ it the integrating variable [Kharkevich, 2009]. Herewith, the obtained amplitude–frequency spectrum characterizes the values of harmonic amplitudes, presented in the signal *f = f(t)*.

We obtain the interrelation between spectrum (1) and harmonic $A\sin(\omega t + \varphi)$, presented in the signal *f = f(t)*, where A and φ are the amplitude and the initial harmonic phase, respectively. With this aim let us substitute the harmonic A sin(ωτ + φ) into Eq. (1) and consider generation of spectrum for this harmonic in time.

Upon substitution of harmonic A sin(ωτ + φ) into Eq. (1) as an analyzed signal *f = f(t)* this equation is as follows:

$\Phi(t,\omega,\varphi) =$

$$= \sqrt{[\int_0^t A\sin(\omega\tau + \varphi)\times\sin(\omega\tau)\, d\tau]^2 + [\int_0^t A\sin(\omega\tau + \varphi)\times\cos(\omega\tau) d\tau]^2} =$$

$$=A\sqrt{[\int_0^t \sin(\omega\tau+\varphi)\times\sin(\omega\tau)\,d\tau]^2+[\int_0^t \sin(\omega\tau+\varphi)\times\cos(\omega\tau)d\tau]^2} \quad (2)$$

Transforming trigonometric expressions presented in Eq. (2) and taking integrals I_1 and I_2 of the transform results of Eqs. (3) and (4) we obtain:

$$\sin(\omega\tau+\varphi)\times\sin(\omega\tau)=(\sin\omega\tau\times\cos\varphi+\cos\omega\tau\times\sin\varphi)\times\sin\omega\tau=$$

$$=sin^2\omega\tau\times\cos\varphi+\frac{1}{2}\sin 2\omega\tau\times\sin\varphi \quad (3)$$

$$\sin(\omega\tau+\varphi)\times\cos(\omega\tau)=(\sin\omega\tau\times\cos\varphi+\cos\omega\tau\times\sin\varphi)\times\cos\omega\tau=$$

$$=\frac{1}{2}\sin 2\,\omega\tau\times\cos\varphi+cos^2\omega\tau\times\sin\varphi \quad (4)$$

$$I_1=\int_0^t[sin^2\,\omega\tau\times cos\varphi+\frac{1}{2}sin2\omega\tau\times sin\varphi]d\tau=$$

$$=cos\varphi\int_0^t sin^2\omega\tau\,d\tau+\frac{1}{2}sin\varphi\int_0^t sin2\omega\tau\,d\tau=$$

$$=\frac{cos\varphi}{\omega}\int_0^t sin^2\omega\tau\,d\omega\tau+\frac{sin\varphi}{4\omega}\int_0^t sin2\omega\tau\,d2\omega\tau=$$

$$=\frac{\cos\varphi}{\omega}\left(\frac{\omega t}{2}-\frac{\sin 2\omega t}{4}\right)+\frac{\sin\varphi}{4\omega}(-\cos 2\omega t+1)=$$

$$=\frac{t}{2}\,cos\varphi-\frac{1}{4\omega}[sin(2\omega t+\varphi)-\sin\varphi]=$$

$$=\frac{t}{2}\cos\varphi-\frac{1}{2\omega}\cos(\omega t+\varphi)\sin\omega t \quad (5)$$

$$I_2=\int_0^t(\frac{1}{2}\sin 2\omega\tau\times\cos\varphi+\cos^2\omega\tau\times\sin\varphi)\,d\tau=$$

$$=\frac{1}{2}\,cos\varphi\int_0^t sin2\omega\tau\,d\tau+sin\varphi\int_0^t cos^2\,\omega\tau\,d\tau=$$

$$=\frac{cos\varphi}{4\omega}\int_0^t sin2\omega\tau\,d2\omega t+\frac{sin\varphi}{\omega}\int_0^t cos^2\,\omega\tau\,d\omega\tau=$$

$$=\frac{\cos\varphi}{4\omega}(-cos2\omega\tau+1)+\frac{\sin\varphi}{\omega}\left(\frac{\omega t}{2}+\frac{\sin 2\omega t}{4}\right)=$$

$$=\frac{t}{2}\,\sin\varphi-\frac{1}{4\omega}[\cos(2\omega t+\varphi)-cos\varphi]=$$

$$=\frac{t}{2}\sin\varphi+\frac{1}{2\omega}\sin(\omega t+\varphi)\sin\omega t \quad (6)$$

By squaring the obtained values of integrals (5) and (6) and summing them we obtain as follows:

$$I_1{}^2 + I_2{}^2 = [\frac{t}{2}\cos\varphi - \frac{1}{2\omega}\cos(\omega t + \varphi)\sin\omega t]^2 +$$

$$+[\frac{t}{2}\sin\varphi + \frac{1}{2\omega}\sin(\omega t + \varphi)\sin\omega t]^2 =$$

$$= \frac{t^2}{4} - \frac{t}{2\omega}\sin\omega t[\cos(\omega t + \varphi)\cos\varphi - \sin(\omega t + \varphi)\sin\varphi] + \frac{sin^2\omega t}{4\omega^2} =$$

$$= \frac{t^2}{4} - \frac{t}{2\omega}\cos(\omega t + 2\varphi)\sin\omega t + \frac{1 - \cos 2\omega t}{8\omega^2} \quad (7)$$

Substituting Eq. (7) into Eq. (2), we obtain:

$$\Phi(t, \omega, \varphi) = A\sqrt{\frac{t^2}{4} - \frac{t}{2\omega}\cos(\omega t + 2\varphi)\sin\omega t + \frac{1-\cos 2\,\omega t}{8\omega^2}} =$$

$$= \frac{At}{2}\sqrt{1 - \frac{2}{\omega t}\cos(\omega t + 2\varphi)\sin\omega t + \frac{1-\cos 2\,\omega t}{2(\omega t)^2}} =$$

$$= \frac{At}{2}\sqrt{1 - \frac{1}{\omega t}[\sin(2\omega t + 2\varphi) - \sin 2\varphi] + \frac{1-\cos 2\omega t}{2(\omega t)^2}} \quad (8)$$

According to Eq. (8) the current amplitude–frequency spectrum upon its generation at frequency ω increases linearly in time with imposition of pulsations with frequency 2ω, which decease in time.

From Eq. (8) we obtain the required Eq. (9), reflecting interrelation between harmonic amplitude and amplitude–frequency spectrum at frequency ω, generated in time on the basis of Eq. (2)

$$A = \frac{2\,\Phi(t,\omega,\varphi)}{t\sqrt{1 - \frac{1}{\omega t}[\sin(2\omega t + 2\varphi) - \sin 2\varphi] + \frac{1-\cos 2\omega t}{2(\omega t)^2}}} \quad (9)$$

RESULTS

The main result is Eq. (9), which, as already mentioned, establishes interrelation between amplitudes of harmonic components and current value of amplitude–frequency spectrum at frequency ω.

Equation (9) is an accurate equation at adopted initial limitations. Thus, if we know current value of spectrum $\Phi(t, \omega, \varphi)$, frequency ω, time of spectrum generation t, and initial phase of harmonic φ, then, using Eq. (9) we can accurately calculate amplitude A of harmonic with frequency ω.

However, in practice it is inconvenient to use Eq. (9). This is stipulated by the fact that the initial phases φ of harmonics in analyzed signals are generally unknown. Thus, in order to determine harmonic amplitudes it would be reasonable to apply approximation variant of Eq. (9) :

$$A = \frac{2\,\Phi(t, \omega, \varphi)}{t} \quad (10)$$

Approximation Eq. (10) is derived from Eq. (9) by exclusion of the constituent $\{-\frac{1}{\omega t}\left[\sin(2\omega t + 2\varphi) - \sin 2\varphi\right] + \frac{1-\cos 2\omega t}{2(\omega t)^2}\}$ from radical expression of Eq. (9).

Upon finding of amplitudes by approximation Eq. (10), the accurate value of amplitude with consideration for possible values of sinusoidal and cosinusoidal components will be in the range of:

$$\frac{2\Phi(t,\omega,\varphi)}{t\sqrt{1+\frac{2}{\omega t}+\frac{1}{(\omega t)^2}}} \leq A \leq \frac{2\Phi(t,\omega,\varphi)}{t\sqrt{1-\frac{2}{\omega t}}} \quad (11)$$

Accordingly, the absolute calculation error of amplitude calculation by Eq. (10) equals to:

$$\Delta A = \frac{2\Phi(t,\omega,\varphi)}{t\sqrt{1-\frac{2}{\omega t}}} - \frac{2\Phi(t,\omega,\varphi)}{t\sqrt{1+\frac{2}{\omega t}+\frac{1}{(\omega t)^2}}} =$$

$$= \frac{2\Phi(t,\omega,\varphi)}{t}\left(\frac{1}{\sqrt{1-\frac{2}{\omega t}}} - \frac{1}{\sqrt{1+\frac{2}{\omega t}+\frac{1}{(\omega t)^2}}}\right), (12)$$

which can be as low as required by selection of variable t.

DISCUSSION

The approach to detection of harmonic amplitudes in analyzed signals is of certain interest. This approach attracts attention because it facilitates generation of working equations with simultaneous indication of their errors.

However, the limitations applied to the researches reduce the field of application of the working equation generated in this approach. Thus, the harmonics with frequencies other then ω according to Eq. (2) can, in principle, influence to some extent on generation of amplitude–frequency spectrum at frequency ω. The methods of numerical integration which will be applied upon calculation of current amplitude–frequency spectrum are characterized by procedural error, which also can influence to some extent on the accuracy of the obtained results.

Therefore, in the course of subsequent researches it would be reasonable to estimate the influence of the mentioned factors on the results.

CONCLUSIONS

This article has been aimed at revealing of interrelation of current amplitude–frequency spectrum and amplitudes of harmonic components in analyzed signals with subsequent derivation of working equation for detection of amplitudes.

Therefore, the following results are obtained:

- The equation, reflecting interrelation between amplitudes of harmonic components in analyzed signals and current amplitude–frequency spectrum, has been obtained with consideration for initial limitations;
- The working equation for calculation of harmonic amplitudes is proposed;
- The error of the working equation has been estimated, which can be as low as required by selection of analysis time.

While performing the researches it has been assumed that determining influence on generation of spectrum at frequency ω is contributed by the harmonic with frequency ω. In addition, the influence of error of numerical calculations on the results has not been taken into account. It is planned to consider for the mentioned factors upon subsequent researches.

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